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# **PyWavelets Documentation**

*Release 1.0.1*

**The PyWavelets Developers**

**Sep 25, 2018**



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PyWavelets is open source wavelet transform software for [Python](#). It combines a simple high level interface with low level C and Cython performance.

PyWavelets is very easy to use and get started with. Just install the package, open the Python interactive shell and type:

```
>>> import pywt
>>> cA, cD = pywt.dwt([1, 2, 3, 4], 'db1')
```

Voilà! Computing wavelet transforms has never been so simple :)

Here is a slightly more involved example of applying a digital wavelet transform to an image:

```
import numpy as np
import matplotlib.pyplot as plt

import pywt
import pywt.data

# Load image
original = pywt.data.camera()

# Wavelet transform of image, and plot approximation and details
titles = ['Approximation', 'Horizontal detail',
         'Vertical detail', 'Diagonal detail']
coeffs2 = pywt.dwt2(original, 'bior1.3')
LL, (LH, HL, HH) = coeffs2
fig = plt.figure(figsize=(12, 3))
for i, a in enumerate([LL, LH, HL, HH]):
    ax = fig.add_subplot(1, 4, i + 1)
    ax.imshow(a, interpolation="nearest", cmap=plt.cm.gray)
    ax.set_title(titles[i], fontsize=10)
    ax.set_xticks([])
    ax.set_yticks([])

fig.tight_layout()
plt.show()
```



The main features of PyWavelets are:

- 1D, 2D and nD Forward and Inverse Discrete Wavelet Transform (DWT and IDWT)
- 1D, 2D and nD Multilevel DWT and IDWT
- 1D, 2D and nD Stationary Wavelet Transform (Undecimated Wavelet Transform)
- 1D and 2D Wavelet Packet decomposition and reconstruction
- 1D Continuous Wavelet Transform
- Computing Approximations of wavelet and scaling functions
- Over 100 [built-in wavelet filters](#) and support for custom wavelets
- Single and double precision calculations
- Real and complex calculations
- Results compatible with Matlab Wavelet Toolbox (TM)



## CHAPTER 2

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### Getting help

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Use [GitHub Issues](#), [StackOverflow](#), or the [PyWavelets discussions group](#) to post your comments or questions.



## CHAPTER 3

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### License

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PyWavelets is a free Open Source software released under the MIT license.



## CHAPTER 4

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### Citing

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If you use PyWavelets in a scientific publication, we would appreciate citations of the project:

Lee G, Gommers R, Wasilewski F, Wohlfahrt K, O’Leary A, Nahrstaedt H, and Contributors, “PyWavelets - Wavelet Transforms in Python”, 2006-, <https://github.com/PyWavelets/pywt> [Online; accessed 2018-MM-DD].



## 5.1 Installing

The latest release, including binary packages for Windows, macOS and Linux, is available for download from [PyPI](#). You can also find source releases at the [Releases Page](#).

You can install PyWavelets with:

```
pip install PyWavelets
```

Users of the [Anaconda](#) Python distribution may wish to obtain pre-built Windows, Intel Linux or macOS / OSX binaries from the main or conda-forge channel:

```
conda install pywavelets
```

Several Linux distributions have their own packages for PyWavelets, but these tend to be moderately out of date. Query your Linux package manager tool for `python-pywavelets`, `python-wavelets`, `python-pywt` or a similar package name.

### 5.1.1 Building from source

The most recent *development* version can be found on GitHub at <https://github.com/PyWavelets/pywt>.

The latest release, is available for download from [PyPI](#) or on the [Releases Page](#).

If you want or need to install from source, you will need a working C compiler (any common one will work) and a recent version of [Cython](#). Navigate to the PyWavelets source code directory (containing `setup.py`) and type:

```
pip install .
```

The requirements needed to build from source are:

- Python 2.7 or  $\geq 3.4$
- Numpy  $\geq 1.9.1$

- `Cython`  $\geq$  0.20.2 (if installing from git, not from a PyPI source release)

To run all the tests for PyWavelets, you will also need to install the [Matplotlib](#) package.

**See also:**

*Development guide* section contains more information on building and installing from source code.

## 5.2 API Reference

### 5.2.1 Wavelets

#### Wavelet families()

`pywt.families(short=True)`

Returns a list of available built-in wavelet families.

Currently the built-in families are:

- Haar (`haar`)
- Daubechies (`db`)
- Symlets (`sym`)
- Coiflets (`coif`)
- Biorthogonal (`bior`)
- Reverse biorthogonal (`rbio`)
- “Discrete” FIR approximation of Meyer wavelet (`dmey`)
- Gaussian wavelets (`gaus`)
- Mexican hat wavelet (`mexh`)
- Morlet wavelet (`morl`)
- Complex Gaussian wavelets (`cgau`)
- Shannon wavelets (`shan`)
- Frequency B-Spline wavelets (`fbsp`)
- Complex Morlet wavelets (`cmor`)

#### Parameters

**short** [bool, optional] Use short names (default: True).

#### Returns

**families** [list] List of available wavelet families.

#### Examples

```

>>> import pywt
>>> pywt.families()
['haar', 'db', 'sym', 'coif', 'bior', 'rbio', 'dmey', 'gaus', 'mexh', 'morl',
 ↪ 'cgau', 'shan', 'fbsp', 'cmor']
>>> pywt.families(short=False)
['Haar', 'Daubechies', 'Symlets', 'Coiflets', 'Biorthogonal', 'Reverse_
 ↪ biorthogonal', 'Discrete Meyer (FIR Approximation)', 'Gaussian', 'Mexican hat_
 ↪ wavelet', 'Morlet wavelet', 'Complex Gaussian wavelets', 'Shannon wavelets',
 ↪ 'Frequency B-Spline wavelets', 'Complex Morlet wavelets']

```

## Built-in wavelets - `wavelist()`

`pywt.wavelist` (*family=None, kind='all'*)

Returns list of available wavelet names for the given family name.

### Parameters

**family** [str, optional] Short family name. If the family name is None (default) then names of all the built-in wavelets are returned. Otherwise the function returns names of wavelets that belong to the given family. Valid names are:

```

'haar', 'db', 'sym', 'coif', 'bior', 'rbio', 'dmey', 'gaus',
'mexh', 'morl', 'cgau', 'shan', 'fbsp', 'cmor'

```

**kind** [{'all', 'continuous', 'discrete'}, optional] Whether to return only wavelet names of discrete or continuous wavelets, or all wavelets. Default is 'all'. Ignored if family is specified.

### Returns

**wavelist** [list of str] List of available wavelet names.

## Examples

```

>>> import pywt
>>> pywt.wavelist('coif')
['coif1', 'coif2', 'coif3', 'coif4', 'coif5', 'coif6', 'coif7', ...
>>> pywt.wavelist(kind='continuous')
['cgau1', 'cgau2', 'cgau3', 'cgau4', 'cgau5', 'cgau6', 'cgau7', ...

```

Custom discrete wavelets are also supported through the *Wavelet* object constructor as described below.

## Wavelet object

**class** `pywt.Wavelet` (*name[, filter\_bank=None]*)

Describes properties of a discrete wavelet identified by the specified wavelet name. For continuous wavelets see `pywt.ContinuousWavelet` instead. In order to use a built-in wavelet the name parameter must be a valid wavelet name from the `pywt.wavelist()` list.

Custom Wavelet objects can be created by passing a user-defined filters set with the `filter_bank` parameter.

### Parameters

- **name** – Wavelet name
- **filter\_bank** – Use a user supplied filter bank instead of a built-in *Wavelet*.

The filter bank object can be a list of four filters coefficients or an object with `filter_bank` attribute, which returns a list of such filters in the following order:

```
[dec_lo, dec_hi, rec_lo, rec_hi]
```

Wavelet objects can also be used as a base filter banks. See section on *using custom wavelets* for more information.

**Example:**

```
>>> import pywt
>>> wavelet = pywt.Wavelet('db1')
```

**name**

Wavelet name.

**short\_name**

Short wavelet name.

**dec\_lo**

Decomposition filter values.

**dec\_hi**

Decomposition filter values.

**rec\_lo**

Reconstruction filter values.

**rec\_hi**

Reconstruction filter values.

**dec\_len**

Decomposition filter length.

**rec\_len**

Reconstruction filter length.

**filter\_bank**

Returns filters list for the current wavelet in the following order:

```
[dec_lo, dec_hi, rec_lo, rec_hi]
```

**inverse\_filter\_bank**

Returns list of reverse wavelet filters coefficients. The mapping from the `filter_coeffs` list is as follows:

```
[rec_lo[::-1], rec_hi[::-1], dec_lo[::-1], dec_hi[::-1]]
```

**short\_family\_name**

Wavelet short family name

**family\_name**

Wavelet family name

**orthogonal**

Set if wavelet is orthogonal

**biorthogonal**

Set if wavelet is biorthogonal

**symmetry**

asymmetric, near symmetric, symmetric

**vanishing\_moments\_psi**

Number of vanishing moments for the wavelet function

**vanishing\_moments\_phi**

Number of vanishing moments for the scaling function

**Example:**

```
>>> def format_array(arr):
...     return "[%s]" % ", ".join(["%.14f" % x for x in arr])

>>> import pywt
>>> wavelet = pywt.Wavelet('db1')
>>> print(wavelet)
Wavelet db1
  Family name:   Daubechies
  Short name:    db
  Filters length: 2
  Orthogonal:    True
  Biorthogonal:  True
  Symmetry:      asymmetric
  DWT:           True
  CWT:           False
>>> print(format_array(wavelet.dec_lo), format_array(wavelet.dec_hi))
[0.70710678118655, 0.70710678118655] [-0.70710678118655, 0.70710678118655]
>>> print(format_array(wavelet.rec_lo), format_array(wavelet.rec_hi))
[0.70710678118655, 0.70710678118655] [0.70710678118655, -0.70710678118655]
```

**Approximating wavelet and scaling functions - Wavelet.wavefun()**

## Wavelet.wavefun(level)

Changed in version 0.2: The time (space) localisation of approximation function points was added.

The `wavefun()` method can be used to calculate approximations of scaling function (`phi`) and wavelet function (`psi`) at the given level of refinement.

For *orthogonal* wavelets returns approximations of scaling function and wavelet function with corresponding x-grid coordinates:

```
[phi, psi, x] = wavelet.wavefun(level)
```

**Example:**

```
>>> import pywt
>>> wavelet = pywt.Wavelet('db2')
>>> phi, psi, x = wavelet.wavefun(level=5)
```

For other (*biorthogonal* but not *orthogonal*) wavelets returns approximations of scaling and wavelet function both for decomposition and reconstruction and corresponding x-grid coordinates:

```
[phi_d, psi_d, phi_r, psi_r, x] = wavelet.wavefun(level)
```

**Example:**

```
>>> import pywt
>>> wavelet = pywt.Wavelet('bior3.5')
>>> phi_d, psi_d, phi_r, psi_r, x = wavelet.wavefun(level=5)
```

**See also:**

You can find live examples of `wavefun()` usage and images of all the built-in wavelets on the [Wavelet Properties Browser](#) page. However, **this website is no longer actively maintained** and does not include every wavelet present in PyWavelets. The precision of the wavelet coefficients at that site is also lower than those included in PyWavelets.

**Using custom wavelets**

PyWavelets comes with a *long list* of the most popular wavelets built-in and ready to use. If you need to use a specific wavelet which is not included in the list it is very easy to do so. Just pass a list of four filters or an object with a `filter_bank` attribute as a `filter_bank` argument to the `Wavelet` constructor.

The filters list, either in a form of a simple Python list or returned via the `filter_bank` attribute, must be in the following order:

- lowpass decomposition filter
- highpass decomposition filter
- lowpass reconstruction filter
- highpass reconstruction filter

just as for the `filter_bank` attribute of the `Wavelet` class.

The Wavelet object created in this way is a standard `Wavelet` instance.

The following example illustrates the way of creating custom Wavelet objects from plain Python lists of filter coefficients and a *filter bank-like* object.

**Example:**

```
>>> import pywt, math
>>> c = math.sqrt(2)/2
>>> dec_lo, dec_hi, rec_lo, rec_hi = [c, c], [-c, c], [c, c], [c, -c]
>>> filter_bank = [dec_lo, dec_hi, rec_lo, rec_hi]
>>> myWavelet = pywt.Wavelet(name="myHaarWavelet", filter_bank=filter_bank)
>>>
>>> class HaarFilterBank(object):
...     @property
...     def filter_bank(self):
...         c = math.sqrt(2)/2
...         dec_lo, dec_hi, rec_lo, rec_hi = [c, c], [-c, c], [c, c], [c, -c]
...         return [dec_lo, dec_hi, rec_lo, rec_hi]
>>> filter_bank = HaarFilterBank()
>>> myOtherWavelet = pywt.Wavelet(name="myHaarWavelet", filter_bank=filter_
↪bank)
```

**ContinuousWavelet object**

**class** `pywt.ContinuousWavelet` (*name*)

Describes properties of a continuous wavelet identified by the specified wavelet name. In order to use a built-in wavelet the name parameter must be a valid wavelet name from the `pywt.wavelist()` list.

**Parameters** `name` – Wavelet name

**Example:**

```
>>> import pywt
>>> wavelet = pywt.ContinuousWavelet('gaus1')
```

**name**  
Continuous Wavelet name.

**short\_family\_name**  
Wavelet short family name

**family\_name**  
Wavelet family name

**orthogonal**  
Set if wavelet is orthogonal

**biorthogonal**  
Set if wavelet is biorthogonal

**complex\_cwt**  
Returns if wavelet is complex

**lower\_bound**  
Set the lower bound of the effective support

**upper\_bound**  
Set the upper bound of the effective support

**center\_frequency**  
Set the center frequency for the shan, fbsp and cmor wavelets

**bandwidth\_frequency**  
Set the bandwidth frequency for the shan, fbsp and cmor wavelets

**fbsp\_order**  
Set the order for the fbsp wavelet

**symmetry**  
asymmetric, near symmetric, symmetric, anti-symmetric

**Example:**

```
>>> import pywt
>>> wavelet = pywt.ContinuousWavelet('gaus1')
>>> print(wavelet)
ContinuousWavelet gaus1
  Family name: Gaussian
  Short name: db
  Symmetry: anti-symmetric
  DWT: False
  CWT: True
  Complex CWT: False
```

**Approximating wavelet functions - `ContinuousWavelet.wavefun()`**

`ContinuousWavelet.wavefun` (*level*, *length* = *None*)

The `wavefun()` method can be used to calculate approximations of scaling function (`psi`) with grid (`x`). The vector length is set by `length`. The vector length can also be defined by `2**level` if `length` is not set.

For `complex_cwt` wavelets returns a complex approximations of wavelet function with corresponding `x`-grid coordinates:

```
[psi, x] = wavelet.wavefun(level)
```

**Example:**

```
>>> import pywt
>>> wavelet = pywt.ContinuousWavelet('gaus1')
>>> psi, x = wavelet.wavefun(level=5)
```

**Approximating wavelet functions - ContinuousWavelet.wavefun()**

`pywt.DiscreteContinuousWavelet(name[, filter_bank = None])`

The `DiscreteContinuousWavelet()` returns a Wavelet or a ContinuousWavelet object depending on the given name.

**Example:**

```
>>> import pywt
>>> wavelet = pywt.DiscreteContinuousWavelet('db1')
>>> print(wavelet)
Wavelet db1
  Family name:  Daubechies
  Short name:   db
  Filters length: 2
  Orthogonal:   True
  Biorthogonal: True
  Symmetry:     asymmetric
  DWT:          True
  CWT:          False
>>> wavelet = pywt.DiscreteContinuousWavelet('gaus1')
>>> print(wavelet)
ContinuousWavelet gaus1
  Family name:  Gaussian
  Short name:   db
  Symmetry:     anti-symmetric
  DWT:          False
  CWT:          True
  Complex CWT:  False
```

**5.2.2 Signal extension modes**

Because the most common and practical way of representing digital signals in computer science is with finite arrays of values, some extrapolation of the input data has to be performed in order to extend the signal before computing the *Discrete Wavelet Transform* using the cascading filter banks algorithm.

Depending on the extrapolation method, significant artifacts at the signal's borders can be introduced during that process, which in turn may lead to inaccurate computations of the *DWT* at the signal's ends.

PyWavelets provides several methods of signal extrapolation that can be used to minimize this negative effect:

- zero - **zero-padding** - signal is extended by adding zero samples:

```
... 0 0 | x1 x2 ... xn | 0 0 ...
```

- constant - **constant-padding** - border values are replicated:

```
... x1 x1 | x1 x2 ... xn | xn xn ...
```

- `symmetric` - **symmetric-padding** - signal is extended by *mirroring* samples. This mode is also known as half-sample symmetric.:

```
... x2 x1 | x1 x2 ... xn | xn xn-1 ...
```

- `reflect` - **reflect-padding** - signal is extended by *reflecting* samples. This mode is also known as whole-sample symmetric.:

```
... x3 x2 | x1 x2 ... xn | xn-1 xn-2 ...
```

- `periodic` - **periodic-padding** - signal is treated as a periodic one:

```
... xn-1 xn | x1 x2 ... xn | x1 x2 ...
```

- `smooth` - **smooth-padding** - signal is extended according to the first derivatives calculated on the edges (straight line)
- `antisymmetric` - **anti-symmetric padding** - signal is extended by *mirroring* and negating samples. This mode is also known as half-sample anti-symmetric:

```
... -x2 -x1 | x1 x2 ... xn | -xn -xn-1 ...
```

- `antireflect` - **anti-symmetric-reflect padding** - signal is extended by *reflecting* anti-symmetrically about the edge samples. This mode is also known as whole-sample anti-symmetric:

```
... (2*x1 - x3) (2*x1 - x2) | x1 x2 ... xn | (2*xn - xn-1) (2*xn - xn-2) ↵
↵ ...
```

*DWT* performed for these extension modes is slightly redundant, but ensures perfect reconstruction. To receive the smallest possible number of coefficients, computations can be performed with the *periodization* mode:

- `periodization` - **periodization** - is like *periodic-padding* but gives the smallest possible number of decomposition coefficients. *IDWT* must be performed with the same mode.

#### Example:

```
>>> import pywt
>>> print (pywt.Modes.modes)
['zero', 'constant', 'symmetric', 'periodic', 'smooth', 'periodization',
 ↵ 'reflect', 'antisymmetric', 'antireflect']
```

The following figure illustrates how a short signal (red) gets extended (black) outside of its original extent. Note that periodization first extends the signal to an even length prior to using periodic boundary conditions.

```
"""A visual illustration of the various signal extension modes supported in
PyWavelets. For efficiency, in the C routines the array is not actually
extended as is done here. This is just a demo for easier visual explanation of
the behavior of the various boundary modes.
```

```
In practice, which signal extension mode is beneficial will depend on the
signal characteristics. For this particular signal, some modes such as
"periodic", "antisymmetric" and "zeros" result in large discontinuities that
would lead to large amplitude boundary coefficients in the detail coefficients
of a discrete wavelet transform.
```

```
"""
```

(continues on next page)

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```

import numpy as np
from matplotlib import pyplot as plt
from pywt._doc_utils import boundary_mode_subplot

# synthetic test signal
x = 5 - np.linspace(-1.9, 1.1, 9)**2

# Create a figure with one subplots per boundary mode
fig, axes = plt.subplots(3, 3, figsize=(10, 6))
plt.subplots_adjust(hspace=0.5)
axes = axes.ravel()
boundary_mode_subplot(x, 'symmetric', axes[0], symw=False)
boundary_mode_subplot(x, 'reflect', axes[1], symw=True)
boundary_mode_subplot(x, 'periodic', axes[2], symw=False)
boundary_mode_subplot(x, 'antisymmetric', axes[3], symw=False)
boundary_mode_subplot(x, 'antireflect', axes[4], symw=True)
boundary_mode_subplot(x, 'periodization', axes[5], symw=False)
boundary_mode_subplot(x, 'smooth', axes[6], symw=False)
boundary_mode_subplot(x, 'constant', axes[7], symw=False)
boundary_mode_subplot(x, 'zeros', axes[8], symw=False)
plt.show()

```

Notice that you can use any of the following ways of passing wavelet and mode parameters:

```

>>> import pywt
>>> (a, d) = pywt.dwt([1,2,3,4,5,6], 'db2', 'smooth')
>>> (a, d) = pywt.dwt([1,2,3,4,5,6], pywt.Wavelet('db2'), pywt.Modes.smooth)

```

**Note:** Extending data in context of PyWavelets does not mean reallocation of the data in the computer's physical memory and copying values, but rather computing the extra values only when they are needed. This feature saves extra memory and CPU resources and helps to avoid page swapping when handling relatively big data arrays on computers with low physical memory.

## Naming Conventions

The correspondence between PyWavelets edge modes and the extension modes available in Matlab's `dwtmode` and numpy's `pad` are tabulated here for reference.

PyWavelets	Matlab	numpy.pad
symmetric	sym, symh	symmetric
reflect	symw	reflect
smooth	spd, sp1	N/A
constant	sp0	edge
zero	zpd	constant, cval=0
periodic	ppd	wrap
periodization	per	N/A
antisymmetric	asym, asymh	N/A
antireflect	asymw	N/A

### 5.2.3 Discrete Wavelet Transform (DWT)

Wavelet transform has recently become a very popular when it comes to analysis, de-noising and compression of signals and images. This section describes functions used to perform single- and multilevel Discrete Wavelet Transforms.

#### Single level `dwt`

`pywt.dwt` (*data*, *wavelet*, *mode*='symmetric', *axis*=-1)  
Single level Discrete Wavelet Transform.

##### Parameters

- data** [array\_like] Input signal
- wavelet** [Wavelet object or name] Wavelet to use
- mode** [str, optional] Signal extension mode, see Modes
- axis: int, optional** Axis over which to compute the DWT. If not given, the last axis is used.

##### Returns

- (cA, cD)** [tuple] Approximation and detail coefficients.

#### Notes

Length of coefficients arrays depends on the selected mode. For all modes except periodization:

$$\text{len}(cA) == \text{len}(cD) == \text{floor}((\text{len}(\text{data}) + \text{wavelet.dec\_len} - 1) / 2)$$

For periodization mode ("per"):

$$\text{len}(cA) == \text{len}(cD) == \text{ceil}(\text{len}(\text{data}) / 2)$$

#### Examples

```
>>> import pywt
>>> (cA, cD) = pywt.dwt([1, 2, 3, 4, 5, 6], 'db1')
>>> cA
array([ 2.12132034,  4.94974747,  7.77817459])
>>> cD
array([-0.70710678, -0.70710678, -0.70710678])
```

See the [signal extension modes](#) section for the list of available options and the `dwt_coeff_len()` function for information on getting the expected result length.

The transform can be performed over one axis of multi-dimensional data. By default this is the last axis. For multi-dimensional transforms see the [2D transforms](#) section.

#### Multilevel decomposition using `wavedec`

`pywt.wavedec` (*data*, *wavelet*, *mode*='symmetric', *level*=None, *axis*=-1)  
Multilevel 1D Discrete Wavelet Transform of data.

##### Parameters

- data: array\_like** Input data

**wavelet** [Wavelet object or name string] Wavelet to use

**mode** [str, optional] Signal extension mode, see *Modes* (default: 'symmetric')

**level** [int, optional] Decomposition level (must be  $\geq 0$ ). If level is None (default) then it will be calculated using the *dwt\_max\_level* function.

**axis: int, optional** Axis over which to compute the DWT. If not given, the last axis is used.

#### Returns

[**cA<sub>n</sub>**, **cD<sub>n</sub>**, **cD<sub>n-1</sub>**, ..., **cD<sub>2</sub>**, **cD<sub>1</sub>**] [list] Ordered list of coefficients arrays where *n* denotes the level of decomposition. The first element (*cA<sub>n</sub>*) of the result is approximation coefficients array and the following elements (*cD<sub>n</sub>* - *cD<sub>1</sub>*) are details coefficients arrays.

#### Examples

```
>>> from pywt import wavedec
>>> coeffs = wavedec([1,2,3,4,5,6,7,8], 'db1', level=2)
>>> cA2, cD2, cD1 = coeffs
>>> cD1
array([-0.70710678, -0.70710678, -0.70710678, -0.70710678])
>>> cD2
array([-2., -2.])
>>> cA2
array([ 5., 13.]
```

### Partial Discrete Wavelet Transform data decomposition `downcoef`

`pywt.downcoef` (*part, data, wavelet, mode='symmetric', level=1*)

Partial Discrete Wavelet Transform data decomposition.

Similar to `pywt.dwt`, but computes only one set of coefficients. Useful when you need only approximation or only details at the given level.

#### Parameters

**part** [str] Coefficients type:

- 'a' - approximations reconstruction is performed
- 'd' - details reconstruction is performed

**data** [array\_like] Input signal.

**wavelet** [Wavelet object or name] Wavelet to use

**mode** [str, optional] Signal extension mode, see *Modes*. Default is 'symmetric'.

**level** [int, optional] Decomposition level. Default is 1.

#### Returns

**coeffs** [ndarray] 1-D array of coefficients.

See also:

*upcoef*

**Maximum decomposition level - `dwt_max_level`, `dwt_n_max_level`**`pywt.dwt_max_level` (*data\_len*, *filter\_len*)

Compute the maximum useful level of decomposition.

**Parameters****data\_len** [int] Input data length.**filter\_len** [int, str or Wavelet] The wavelet filter length. Alternatively, the name of a discrete wavelet or a Wavelet object can be specified.**Returns****max\_level** [int] Maximum level.**Notes**

The rationale for the choice of levels is the maximum level where at least one coefficient in the output is uncorrupted by edge effects caused by signal extension. Put another way, decomposition stops when the signal becomes shorter than the FIR filter length for a given wavelet. This corresponds to:

$$\text{max\_level} = \left\lfloor \log_2 \left( \frac{\text{data\_len}}{\text{filter\_len} - 1} \right) \right\rfloor$$

**Examples**

```
>>> import pywt
>>> w = pywt.Wavelet('sym5')
>>> pywt.dwt_max_level(data_len=1000, filter_len=w.dec_len)
6
>>> pywt.dwt_max_level(1000, w)
6
>>> pywt.dwt_max_level(1000, 'sym5')
6
```

`pywt.dwt_n_max_level` (*shape*, *wavelet*, *axes=None*)

Compute the maximum level of decomposition for n-dimensional data.

This returns the maximum number of levels of decomposition suitable for use with *wavedec*, *wavedec2* or *wavedecn*.

**Parameters****shape** [sequence of ints] Input data shape.**wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.**axes** [sequence of ints, optional] Axes over which to compute the DWT. Axes may not be repeated.**Returns****level** [int] Maximum level.

## Notes

The level returned is the smallest *dwt\_max\_level* over all axes.

## Examples

```
>>> import pywt
>>> pywt.dwt_max_level((64, 32), 'db2')
3
```

## Result coefficients length - `dwt_coeff_len`

`pywt.dwt_coeff_len(data_len, filter_len, mode='symmetric')`

Returns length of dwt output for given data length, filter length and mode

### Parameters

**data\_len** [int] Data length.

**filter\_len** [int] Filter length.

**mode** [str, optional (default: 'symmetric')] Signal extension mode, see Modes

### Returns

**len** [int] Length of dwt output.

## Notes

For all modes except periodization:

```
len(cA) == len(cD) == floor((len(data) + wavelet.dec_len - 1) / 2)
```

for periodization mode ("per"):

```
len(cA) == len(cD) == ceil(len(data) / 2)
```

Based on the given input data length (`data_len`), wavelet decomposition filter length (`filter_len`) and *signal extension mode*, the `dwt_coeff_len()` function calculates the length of the resulting coefficients arrays that would be created while performing `dwt()` transform.

`filter_len` can be either an `int` or `Wavelet` object for convenience.

## 5.2.4 Inverse Discrete Wavelet Transform (IDWT)

### Single level `idwt`

`pywt.idwt(cA, cD, wavelet, mode='symmetric', axis=-1)`

Single level Inverse Discrete Wavelet Transform.

### Parameters

**cA** [array\_like or None] Approximation coefficients. If None, will be set to array of zeros with same shape as `cD`.

**cD** [array\_like or None] Detail coefficients. If None, will be set to array of zeros with same shape as *cA*.

**wavelet** [Wavelet object or name] Wavelet to use

**mode** [str, optional (default: 'symmetric')] Signal extension mode, see Modes

**axis: int, optional** Axis over which to compute the inverse DWT. If not given, the last axis is used.

### Returns

**rec: array\_like** Single level reconstruction of signal from given coefficients.

### Examples

```
>>> import pywt
>>> (cA, cD) = pywt.dwt([1,2,3,4,5,6], 'db2', 'smooth')
>>> pywt.idwt(cA, cD, 'db2', 'smooth')
array([ 1.,  2.,  3.,  4.,  5.,  6.]
```

One of the neat features of *idwt* is that one of the *cA* and *cD* arguments can be set to None. In that situation the reconstruction will be performed using only the other one. Mathematically speaking, this is equivalent to passing a zero-filled array as one of the arguments.

```
>>> (cA, cD) = pywt.dwt([1,2,3,4,5,6], 'db2', 'smooth')
>>> A = pywt.idwt(cA, None, 'db2', 'smooth')
>>> D = pywt.idwt(None, cD, 'db2', 'smooth')
>>> A + D
array([ 1.,  2.,  3.,  4.,  5.,  6.]
```

### Multilevel reconstruction using *waverec*

`pywt.waverec` (*coeffs*, *wavelet*, *mode='symmetric'*, *axis=-1*)  
Multilevel 1D Inverse Discrete Wavelet Transform.

#### Parameters

**coeffs** [array\_like] Coefficients list [*cAn*, *cDn*, *cDn-1*, ..., *cD2*, *cD1*]

**wavelet** [Wavelet object or name string] Wavelet to use

**mode** [str, optional] Signal extension mode, see *Modes* (default: 'symmetric')

**axis: int, optional** Axis over which to compute the inverse DWT. If not given, the last axis is used.

### Notes

It may sometimes be desired to run *waverec* with some sets of coefficients omitted. This can best be done by setting the corresponding arrays to zero arrays of matching shape and dtype. Explicitly removing list entries or setting them to None is not supported.

Specifically, to ignore detail coefficients at level 2, one could do:

```
coeffs[-2] == np.zeros_like(coeffs[-2])
```

## Examples

```
>>> import pywt
>>> coeffs = pywt.wavedec([1,2,3,4,5,6,7,8], 'db1', level=2)
>>> pywt.waverec(coeffs, 'db1')
array([ 1.,  2.,  3.,  4.,  5.,  6.,  7.,  8.])
```

## Direct reconstruction with upcoef

`pywt.upcoef` (*part*, *coeffs*, *wavelet*, *level=1*, *take=0*)

Direct reconstruction from coefficients.

### Parameters

**part** [str] Coefficients type: \* 'a' - approximations reconstruction is performed \* 'd' - details reconstruction is performed

**coeffs** [array\_like] Coefficients array to reconstruct

**wavelet** [Wavelet object or name] Wavelet to use

**level** [int, optional] Multilevel reconstruction level. Default is 1.

**take** [int, optional] Take central part of length equal to 'take' from the result. Default is 0.

### Returns

**rec** [ndarray] 1-D array with reconstructed data from coefficients.

See also:

[\*downcoef\*](#)

## Examples

```
>>> import pywt
>>> data = [1,2,3,4,5,6]
>>> (cA, cD) = pywt.dwt(data, 'db2', 'smooth')
>>> pywt.upcoef('a', cA, 'db2') + pywt.upcoef('d', cD, 'db2')
array([-0.25          , -0.4330127  ,  1.          ,  2.          ,  3.          ,
        4.          ,  5.          ,  6.          ,  1.78589838, -1.03108891])
>>> n = len(data)
>>> pywt.upcoef('a', cA, 'db2', take=n) + pywt.upcoef('d', cD, 'db2', take=n)
array([ 1.,  2.,  3.,  4.,  5.,  6.])
```

## 5.2.5 Overview of multilevel wavelet decompositions

There are a number of different ways a wavelet decomposition can be performed for multiresolution analysis of n-dimensional data. Here we will review the three approaches currently implemented in PyWavelets. 2D cases are illustrated, but each of the approaches extends to the n-dimensional case in a straightforward manner.

### Multilevel Discrete Wavelet Transform

The most common approach to the multilevel discrete wavelet transform involves further decomposition of only the approximation subband at each subsequent level. This is also sometimes referred to as the Mallat decomposition

[Mall89]. In 2D, the discrete wavelet transform produces four sets of coefficients corresponding to the four possible combinations of the wavelet decomposition filters over the two separate axes. (In  $n$ -dimensions, there are  $2^{*n}$  sets of coefficients). For subsequent levels of decomposition, only the approximation coefficients (the lowpass subband) are further decomposed.

In PyWavelets, this decomposition is implemented for  $n$ -dimensional data by `wavedecn()` and the inverse by `waverecn()`. 1D and 2D versions of these routines also exist. It is illustrated in the figure below. The top row indicates the coefficient names as used by `wavedec2()` after each level of decomposition. The bottom row shows wavelet coefficients for the cameraman image (with each subband independently normalized for easier visualization).

```
import numpy as np
import pywt
from matplotlib import pyplot as plt
from pywt._doc_utils import wavedec2_keys, draw_2d_wp_basis

x = pywt.data.camera().astype(np.float32)
shape = x.shape

max_lev = 3      # how many levels of decomposition to draw
label_levels = 3 # how many levels to explicitly label on the plots

fig, axes = plt.subplots(2, 4, figsize=[14, 8])
for level in range(0, max_lev + 1):
    if level == 0:
        # show the original image before decomposition
        axes[0, 0].set_axis_off()
        axes[1, 0].imshow(x, cmap=plt.cm.gray)
        axes[1, 0].set_title('Image')
        axes[1, 0].set_axis_off()
        continue

    # plot subband boundaries of a standard DWT basis
    draw_2d_wp_basis(shape, wavedec2_keys(level), ax=axes[0, level],
                    label_levels=label_levels)
    axes[0, level].set_title('{} level\ndecomposition'.format(level))

    # compute the 2D DWT
    c = pywt.wavedec2(x, 'db2', mode='periodization', level=level)
    # normalize each coefficient array independently for better visibility
    c[0] /= np.abs(c[0]).max()
    for detail_level in range(level):
        c[detail_level + 1] = [d/np.abs(d).max() for d in c[detail_level + 1]]
    # show the normalized coefficients
    arr, slices = pywt.coeffs_to_array(c)
    axes[1, level].imshow(arr, cmap=plt.cm.gray)
    axes[1, level].set_title('Coefficients\n({} level)'.format(level))
    axes[1, level].set_axis_off()

plt.tight_layout()
plt.show()
```

It can be seen that many of the coefficients are near zero (gray). This ability of the wavelet transform to sparsely represent natural images is a key property that makes it desirable in applications such as image compression and restoration.

## Fully Seperable Discrete Wavelet Transform

An alternative decomposition results in first fully decomposing one axis of the data prior to moving onto each additional axis in turn. This is illustrated for the 2D case in the upper right panel of the figure below. This approach has a factor of two higher computational cost as compared to the Mallat approach, but has advantages in compactly representing anisotropic data. A demo of this is [available](#)).

This form of the DWT is also sometimes referred to as the tensor wavelet transform or the hyperbolic wavelet transform. In PyWavelets it is implemented for n-dimensional data by `fswavedecn()` and the inverse by `fswaverecn()`.

## Wavelet Packet Transform

Another possible choice is to apply additional levels of decomposition to all wavelet subbands from the first level as opposed to only the approximation subband. This is known as the wavelet packet transform and is illustrated in 2D in the lower left panel of the figure. It is also possible to only perform any subset of the decompositions, resulting in a wide number of potential wavelet packet bases. An arbitrary example is shown in the lower right panel of the figure below.

A further description is available in the *wavelet packet documentation*.

For the wavelet packets, the plots below use “natural” ordering for simplicity, but this does not directly match the “frequency” ordering for these wavelet packets. It is possible to rearrange the coefficients into frequency ordering (see the `get_level` method of *WaveletPacket2D* and [\[Wick94\]](#) for more details).

```

from itertools import product
import numpy as np
from matplotlib import pyplot as plt
from pywt._doc_utils import (wavedec_keys, wavedec2_keys, draw_2d_wp_basis,
                             draw_2d_fswavedecn_basis)

shape = (512, 512)

max_lev = 4      # how many levels of decomposition to draw
label_levels = 2 # how many levels to explicitly label on the plots

if False:
    fig, axes = plt.subplots(1, 4, figsize=[16, 4])
    axes = axes.ravel()
else:
    fig, axes = plt.subplots(2, 2, figsize=[8, 8])
    axes = axes.ravel()

# plot a 5-level standard DWT basis
draw_2d_wp_basis(shape, wavedec2_keys(max_lev), ax=axes[0],
                 label_levels=label_levels)
axes[0].set_title('wavedec2 ({} level)'.format(max_lev))

# plot for the fully separable case
draw_2d_fswavedecn_basis(shape, max_lev, ax=axes[1], label_levels=label_levels)
axes[1].set_title('fswavedecn ({} level)'.format(max_lev))

# get all keys corresponding to a full wavelet packet decomposition
wp_keys = list(product(['a', 'd', 'h', 'v'], repeat=max_lev))
draw_2d_wp_basis(shape, wp_keys, ax=axes[2])
axes[2].set_title('wavelet packet\n(full: {} level)'.format(max_lev))

```

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```
# plot an example of a custom wavelet packet basis
keys = ['aaaa', 'aaad', 'aaah', 'aaav', 'aad', 'aah', 'aava', 'aavd',
        'aavh', 'aavv', 'ad', 'ah', 'ava', 'avd', 'avh', 'avv', 'd', 'h',
        'vaa', 'vad', 'vah', 'vav', 'vd', 'vh', 'vv']
draw_2d_wp_basis(shape, keys, ax=axes[3], label_levels=label_levels)
axes[3].set_title('wavelet packet\n(custom)'.format(max_lev))

plt.tight_layout()
plt.show()
```

## References

### 5.2.6 2D Forward and Inverse Discrete Wavelet Transform

#### Single level `dwt2`

`pywt.dwt2` (*data*, *wavelet*, *mode*=*'symmetric'*, *axes*=(-2, -1))  
2D Discrete Wavelet Transform.

#### Parameters

**data** [array\_like] 2D array with input data

**wavelet** [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.

**mode** [str or 2-tuple of strings, optional] Signal extension mode, see Modes (default: *'symmetric'*). This can also be a tuple of modes specifying the mode to use on each axis in *axes*.

**axes** [2-tuple of ints, optional] Axes over which to compute the DWT. Repeated elements mean the DWT will be performed multiple times along these axes.

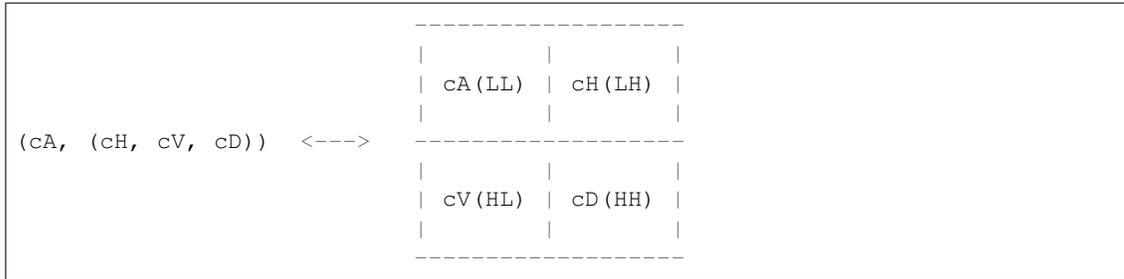
#### Returns

(**cA**, (**cH**, **cV**, **cD**)) [tuple] Approximation, horizontal detail, vertical detail and diagonal detail coefficients respectively. Horizontal refers to array axis 0 (or *axes*[0] for user-specified *axes*).

#### Examples

```
>>> import numpy as np
>>> import pywt
>>> data = np.ones((4,4), dtype=np.float64)
>>> coeffs = pywt.dwt2(data, 'haar')
>>> cA, (cH, cV, cD) = coeffs
>>> cA
array([[ 2.,  2.],
       [ 2.,  2.]])
>>> cV
array([[ 0.,  0.],
       [ 0.,  0.]])
```

The relation to the other common data layout where all the approximation and details coefficients are stored in one big 2D array is as follows:



PyWavelets does not follow this pattern because of pure practical reasons of simple access to particular type of the output coefficients.

### Single level `idwt2`

`pywt.idwt2` (*coeffs*, *wavelet*, *mode*='symmetric', *axes*=(-2, -1))  
 2-D Inverse Discrete Wavelet Transform.

Reconstructs data from coefficient arrays.

#### Parameters

- coeffs** [tuple] (cA, (cH, cV, cD)) A tuple with approximation coefficients and three details coefficients 2D arrays like from `dwt2`. If any of these components are set to `None`, it will be treated as zeros.
- wavelet** [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.
- mode** [str or 2-tuple of strings, optional] Signal extension mode, see Modes (default: 'symmetric'). This can also be a tuple of modes specifying the mode to use on each axis in *axes*.
- axes** [2-tuple of ints, optional] Axes over which to compute the IDWT. Repeated elements mean the IDWT will be performed multiple times along these axes.

### Examples

```
>>> import numpy as np
>>> import pywt
>>> data = np.array([[1,2], [3,4]], dtype=np.float64)
>>> coeffs = pywt.dwt2(data, 'haar')
>>> pywt.idwt2(coeffs, 'haar')
array([[ 1.,  2.],
       [ 3.,  4.]])
```

### 2D multilevel decomposition using `wavedec2`

`pywt.wavedec2` (*data*, *wavelet*, *mode*='symmetric', *level*=None, *axes*=(-2, -1))  
 Multilevel 2D Discrete Wavelet Transform.

#### Parameters

- data** [ndarray] 2D input data

**wavelet** [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.

**mode** [str or 2-tuple of str, optional] Signal extension mode, see *Modes* (default: 'symmetric'). This can also be a tuple containing a mode to apply along each axis in *axes*.

**level** [int, optional] Decomposition level (must be  $\geq 0$ ). If level is None (default) then it will be calculated using the *dwt\_max\_level* function.

**axes** [2-tuple of ints, optional] Axes over which to compute the DWT. Repeated elements are not allowed.

### Returns

[**cAn**, (**cHn**, **cVn**, **cDn**), ... (**cH1**, **cV1**, **cD1**)] [list] Coefficients list. For user-specified *axes*, *cH\** corresponds to *axes*[0] while *cV\** corresponds to *axes*[1]. The first element returned is the approximation coefficients for the *n*th level of decomposition. Remaining elements are tuples of detail coefficients in descending order of decomposition level. (i.e. *cH1* are the horizontal detail coefficients at the first level)

### Examples

```
>>> import pywt
>>> import numpy as np
>>> coeffs = pywt.wavedec2(np.ones((4,4)), 'db1')
>>> # Levels:
>>> len(coeffs)-1
2
>>> pywt.waverec2(coeffs, 'db1')
array([[ 1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.]])
```

### 2D multilevel reconstruction using `waverec2`

`pywt.waverec2` (*coeffs*, *wavelet*, *mode*='symmetric', *axes*=(-2, -1))

Multilevel 2D Inverse Discrete Wavelet Transform.

**coeffs** [list or tuple] Coefficients list [**cAn**, (**cHn**, **cVn**, **cDn**), ... (**cH1**, **cV1**, **cD1**)]

**wavelet** [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.

**mode** [str or 2-tuple of str, optional] Signal extension mode, see *Modes* (default: 'symmetric'). This can also be a tuple containing a mode to apply along each axis in *axes*.

**axes** [2-tuple of ints, optional] Axes over which to compute the IDWT. Repeated elements are not allowed.

### Returns

2D array of reconstructed data.

### Notes

It may sometimes be desired to run *waverec2* with some sets of coefficients omitted. This can best be done by setting the corresponding arrays to zero arrays of matching shape and dtype. Explicitly removing list or tuple

entries or setting them to None is not supported.

Specifically, to ignore all detail coefficients at level 2, one could do:

```
coeffs[-2] == tuple([np.zeros_like(v) for v in coeffs[-2]])
```

## Examples

```
>>> import pywt
>>> import numpy as np
>>> coeffs = pywt.wavedec2(np.ones((4,4)), 'db1')
>>> # Levels:
>>> len(coeffs)-1
2
>>> pywt.waverec2(coeffs, 'db1')
array([[ 1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.]])
```

## 2D coordinate conventions

The labels for “horizontal” and “vertical” used by `dwt2` and `idwt2` follow the common mathematical convention that coordinate axis 0 is horizontal while axis 1 is vertical:

```
dwt2, idwt2 convention
-----
axis 1 ^
      |
      |
      |----->
      axis 0
```

Note that this is different from another common convention used in computer graphics and image processing (e.g. by `matplotlib`’s `imshow` and functions in `scikit-image`). In those packages axis 0 is a vertical axis and axis 1 is horizontal as follows:

```
imshow convention
-----
                axis 1
                |----->
                |
                |
axis 0 v
```

## 5.2.7 nD Forward and Inverse Discrete Wavelet Transform

---

`_multilevel`Multilevel 1D and 2D Discrete Wavelet Transform and Inverse Discrete Wavelet Transform.

---

### Single level - `dwt_n`

`pywt.dwt_n` (*data*, *wavelet*, *mode='symmetric'*, *axes=None*)  
Single-level n-dimensional Discrete Wavelet Transform.

#### Parameters

**data** [array\_like] n-dimensional array with input data.

**wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.

**mode** [str or tuple of string, optional] Signal extension mode used in the decomposition, see Modes (default: 'symmetric'). This can also be a tuple of modes specifying the mode to use on each axis in *axes*.

**axes** [sequence of ints, optional] Axes over which to compute the DWT. Repeated elements mean the DWT will be performed multiple times along these axes. A value of `None` (the default) selects all axes.

Axes may be repeated, but information about the original size may be lost if it is not divisible by  $2^{**nrepeats}$ . The reconstruction will be larger, with additional values derived according to the *mode* parameter. `pywt.wavedecn` should be used for multilevel decomposition.

#### Returns

**coeffs** [dict] Results are arranged in a dictionary, where key specifies the transform type on each dimension and value is a n-dimensional coefficients array.

For example, for a 2D case the result will look something like this:

```
{ 'aa': <coeffs> # A(LL) - approx. on 1st dim, approx. on 2nd dim
  'ad': <coeffs> # V(LH) - approx. on 1st dim, det. on 2nd dim
  'da': <coeffs> # H(HL) - det. on 1st dim, approx. on 2nd dim
  'dd': <coeffs> # D(HH) - det. on 1st dim, det. on 2nd dim
}
```

For user-specified *axes*, the order of the characters in the dictionary keys map to the specified *axes*.

### Single level - `idwt_n`

`pywt.idwt_n` (*coeffs*, *wavelet*, *mode='symmetric'*, *axes=None*)  
Single-level n-dimensional Inverse Discrete Wavelet Transform.

#### Parameters

**coeffs: dict** Dictionary as in output of `dwt_n`. Missing or `None` items will be treated as zeros.

**wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.

**mode** [str or list of string, optional] Signal extension mode used in the decomposition, see Modes (default: 'symmetric'). This can also be a tuple of modes specifying the mode to use on each axis in *axes*.

**axes** [sequence of ints, optional] Axes over which to compute the IDWT. Repeated elements mean the IDWT will be performed multiple times along these axes. A value of `None` (the default) selects all axes.

For the most accurate reconstruction, the axes should be provided in the same order as they were provided to `dwt.n`.

### Returns

**data:** `ndarray` Original signal reconstructed from input data.

## Multilevel decomposition - `wavedecn`

`pywt.wavedecn` (*data*, *wavelet*, *mode='symmetric'*, *level=None*, *axes=None*)  
Multilevel nD Discrete Wavelet Transform.

### Parameters

**data** [`ndarray`] nD input data

**wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.

**mode** [str or tuple of str, optional] Signal extension mode, see *Modes* (default: 'symmetric'). This can also be a tuple containing a mode to apply along each axis in *axes*.

**level** [int, optional] Decomposition level (must be  $\geq 0$ ). If level is `None` (default) then it will be calculated using the `dwt_max_level` function.

**axes** [sequence of ints, optional] Axes over which to compute the DWT. Axes may not be repeated. The default is `None`, which means transform all axes (`axes = range(data.ndim)`).

### Returns

[*cAn*, {*details\_level\_n*}, ... {*details\_level\_1*}] [list] Coefficients list. Coefficients are listed in descending order of decomposition level. *cAn* are the approximation coefficients at level *n*. Each *details\_level\_i* element is a dictionary containing detail coefficients at level *i* of the decomposition. As a concrete example, a 3D decomposition would have the following set of keys in each *details\_level\_i* dictionary:

```
{'aad', 'ada', 'daa', 'add', 'dad', 'dda', 'ddd'}
```

where the order of the characters in each key map to the specified *axes*.

## Examples

```
>>> import numpy as np
>>> from pywt import wavedecn, waverecn
>>> coeffs = wavedecn(np.ones((4, 4, 4)), 'db1')
>>> # Levels:
>>> len(coeffs)-1
2
>>> waverecn(coeffs, 'db1')
array([[[ 1.,  1.,  1.,  1.],
        [ 1.,  1.,  1.,  1.],
        [ 1.,  1.,  1.,  1.],
        [ 1.,  1.,  1.,  1.]],
```

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```

[[ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.]],
 [[ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.]],
 [[ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.]]])

```

### Multilevel reconstruction - `waverecn`

`pywt.waverecn` (*coeffs*, *wavelet*, *mode*='symmetric', *axes*=None)

Multilevel nD Inverse Discrete Wavelet Transform.

**coeffs** [array\_like] Coefficients list [*cAn*, {*details\_level\_n*}, ... {*details\_level\_1*}]

**wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.

**mode** [str or tuple of str, optional] Signal extension mode, see *Modes* (default: 'symmetric'). This can also be a tuple containing a mode to apply along each axis in *axes*.

**axes** [sequence of ints, optional] Axes over which to compute the IDWT. Axes may not be repeated.

#### Returns

**nD array of reconstructed data.**

#### Notes

It may sometimes be desired to run *waverecn* with some sets of coefficients omitted. This can best be done by setting the corresponding arrays to zero arrays of matching shape and dtype. Explicitly removing list or dictionary entries or setting them to None is not supported.

Specifically, to ignore all detail coefficients at level 2, one could do:

```
coeffs[-2] = {k: np.zeros_like(v) for k, v in coeffs[-2].items() }
```

#### Examples

```

>>> import numpy as np
>>> from pywt import wavedecn, waverecn
>>> coeffs = wavedecn(np.ones((4, 4, 4)), 'db1')
>>> # Levels:
>>> len(coeffs)-1
2
>>> waverecn(coeffs, 'db1')
array([[[ 1., 1., 1., 1.],
         [ 1., 1., 1., 1.],

```

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```
[ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.]],
 [[ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.]],
 [[ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.]],
 [[ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.],
 [ 1., 1., 1., 1.]]])
```

### Multilevel fully separable decomposition - `fswavedecn`

`pywt.fswavedecn` (*data*, *wavelet*, *mode*='symmetric', *levels*=None, *axes*=None)

Fully Separable Wavelet Decomposition.

This is a variant of the multilevel discrete wavelet transform where all levels of decomposition are performed along a single axis prior to moving onto the next axis. Unlike in `wavedecn`, the number of levels of decomposition are not required to be the same along each axis which can be a benefit for anisotropic data.

#### Parameters

**data**: `array_like` Input data

**wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in `axes`.

**mode** [str or tuple of str, optional] Signal extension mode, see *Modes* (default: 'symmetric'). This can also be a tuple containing a mode to apply along each axis in `axes`.

**levels** [int or sequence of ints, optional] Decomposition levels along each axis (must be  $\geq 0$ ). If an integer is provided, the same number of levels are used for all axes. If `levels` is None (default), `dwt_max_level` will be used to compute the maximum number of levels possible for each axis.

**axes** [sequence of ints, optional] Axes over which to compute the transform. Axes may not be repeated. The default is to transform along all axes.

#### Returns

**fswavedecn\_result** [FswavedecnResult object] Contains the wavelet coefficients, slice objects to allow obtaining the coefficients per detail or approximation level, and more. See *FswavedecnResult* for details.

See also:

[`fswaverecn`](#) inverse of `fswavedecn`

#### Notes

This transformation has been variously referred to as the (fully) separable wavelet transform (e.g. refs [1], [3]), the tensor-product wavelet ([2]) or the hyperbolic wavelet transform ([4]). It is well suited to data with anisotropic smoothness.

In [2] it was demonstrated that fully separable transform performs at least as well as the DWT for image compression. Computation time is a factor 2 larger than that for the DWT.

## References

[1], [2], [3], [4]

## Examples

```
>>> from pywt import fswavedecn
>>> fs_result = fswavedecn(np.ones((32, 32)), 'sym2', levels=(1, 3))
>>> print(fs_result.detail_keys())
[(0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3)]
>>> approx_coeffs = fs_result.approx
>>> detail_1_2 = fs_result[(1, 2)]
```

## Multilevel fully separable reconstruction - `fswaverecn`

`pywt.fswaverecn` (*fswavedecn\_result*)

Fully Separable Inverse Wavelet Reconstruction.

### Parameters

**fswavedecn\_result** [FswavedecnResult object] FswavedecnResult object from `fswavedecn`.

### Returns

**reconstructed** [ndarray] Array of reconstructed data.

See also:

*fswavedecn* inverse of `fswaverecn`

## Notes

This transformation has been variously referred to as the (fully) separable wavelet transform (e.g. refs [1], [3]), the tensor-product wavelet ([2]) or the hyperbolic wavelet transform ([4]). It is well suited to data with anisotropic smoothness.

In [2] it was demonstrated that the fully separable transform performs at least as well as the DWT for image compression. Computation time is a factor 2 larger than that for the DWT.

## References

[1], [2], [3], [4]

## Multilevel fully separable reconstruction coeffs - `FswavedecnResult`

**class** `pywt.FswavedecnResult` (*coeffs, coeff\_slices, wavelets, mode\_enums, axes*)

Object representing fully separable wavelet transform coefficients.

### Parameters

**coeffs** [ndarray] The coefficient array.

**coeff\_slices** [list] List of slices corresponding to each detail or approximation coefficient array.

**wavelets** [list of pywt.DiscreteWavelet objects] The wavelets used. Will be a list with length equal to `len(axes)`.

**mode\_enums** [list of int] The border modes used. Will be a list with length equal to `len(axes)`.

**axes** [tuple of int] The set of axes over which the transform was performed.

#### Attributes

**approx** ndarray: The approximation coefficients.

**axes** List of str: The axes the transform was performed along.

**coeff\_slices** List: List of coefficient slices.

**coeffs** ndarray: All coefficients stacked into a single array.

**levels** List of int: Levels of decomposition along each transformed axis.

**modes** List of str: The border mode used along each transformed axis.

**ndim** int: Number of data dimensions.

**ndim\_transform** int: Number of axes transformed.

**wavelet\_names** List of pywt.DiscreteWavelet: wavelet for each transformed axis.

**wavelets** List of pywt.DiscreteWavelet: wavelet for each transformed axis.

#### Methods

---

<code>detail_keys()</code>	Return a list of all detail coefficient keys.
----------------------------	---

---

## 5.2.8 Handling DWT Coefficients

Convenience routines are available for converting the outputs of the multilevel dwt functions (`wavedec`, `wavedec2` and `wavedecn`) to and from a single, concatenated coefficient array.

### Concatenating all coefficients into a single n-d array

`pywt.coeffs_to_array` (*coeffs*, *padding=0*, *axes=None*)  
 Arrange a wavelet coefficient list from *wavedecn* into a single array.

#### Parameters

**coeffs** [array-like] dictionary of wavelet coefficients as returned by `pywt.wavedecn`

**padding** [float or None, optional] If None, raise an error if the coefficients cannot be tightly packed.

**axes** [sequence of ints, optional] Axes over which the DWT that created *coeffs* was performed. The default value of None corresponds to all axes.

#### Returns

**coeff\_arr** [array-like] Wavelet transform coefficient array.

**coeff\_slices** [list] List of slices corresponding to each coefficient. As a 2D example, `coeff_arr[coeff_slices[1]['dd']]` would extract the first level detail coefficients from `coeff_arr`.

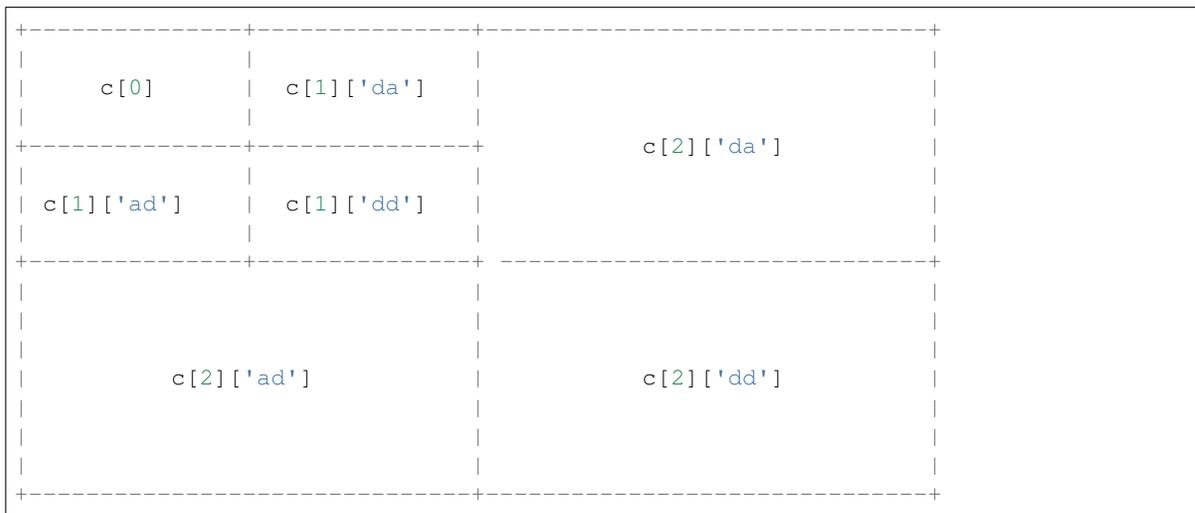
**See also:**

[`array\_to\_coeffs`](#) the inverse of `coeffs_to_array`

## Notes

Assume a 2D coefficient dictionary, `c`, from a two-level transform.

Then all 2D coefficients will be stacked into a single larger 2D array as follows:



## Examples

```
>>> import pywt
>>> cam = pywt.data.camera()
>>> coeffs = pywt.wavedecn(cam, wavelet='db2', level=3)
>>> arr, coeff_slices = pywt.coeffs_to_array(coeffs)
```

## Splitting concatenated coefficient array back into its components

`pywt.array_to_coeffs(arr, coeff_slices, output_format='wavedecn')`

Convert a combined array of coefficients back to a list compatible with `waverecn`.

### Parameters

**arr** [array-like] An array containing all wavelet coefficients. This should have been generated via `coeffs_to_array`.

**coeff\_slices** [list of tuples] List of slices corresponding to each coefficient as obtained from `array_to_coeffs`.

**output\_format** [{ 'wavedec', 'wavedec2', 'wavedecn' }] Make the form of the coefficients compatible with this type of multilevel transform.

### Returns

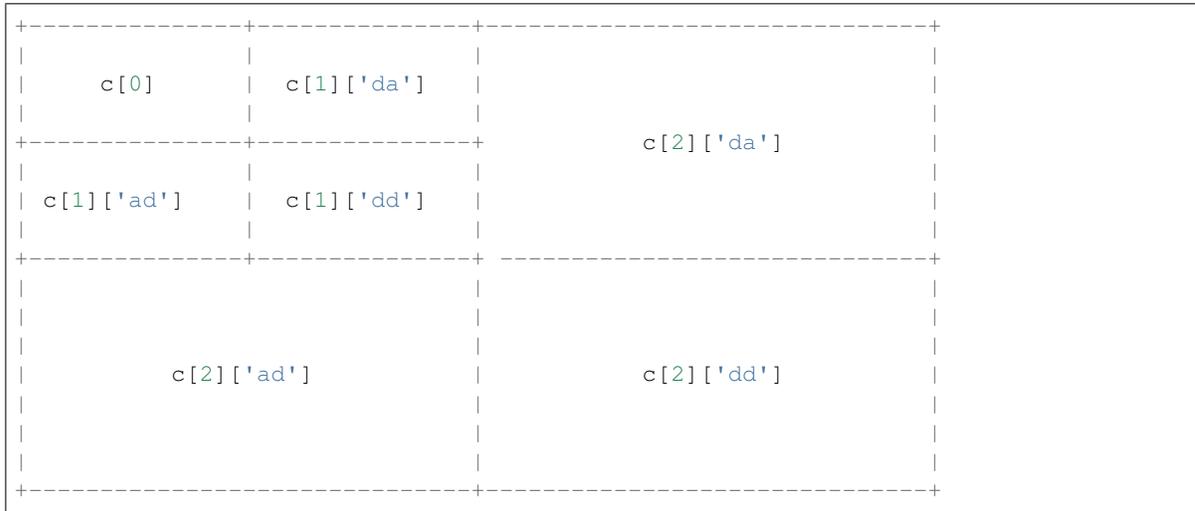
**coeffs:** array-like Wavelet transform coefficient array.

See also:

[\*coeffs\\_to\\_array\*](#) the inverse of [\*array\\_to\\_coeffs\*](#)

## Notes

A single large array containing all coefficients will have subsets stored, into a *waverecn* list, *c*, as indicated below:



## Examples

```
>>> import pywt
>>> from numpy.testing import assert_array_almost_equal
>>> cam = pywt.data.camera()
>>> coeffs = pywt.wavedecn(cam, wavelet='db2', level=3)
>>> arr, coeff_slices = pywt.coeffs_to_array(coeffs)
>>> coeffs_from_arr = pywt.array_to_coeffs(arr, coeff_slices)
>>> cam_recon = pywt.waverecn(coeffs_from_arr, wavelet='db2')
>>> assert_array_almost_equal(cam, cam_recon)
```

## Raveling and unraveling coefficients to/from a 1D array

`pywt.ravel_coeffs` (*coeffs*, *axes=None*)

Ravel a set of multilevel wavelet coefficients into a single 1D array.

### Parameters

**coeffs** [array-like] A list of multilevel wavelet coefficients as returned by *wavedec*, *wavedec2* or *wavedecn*.

**axes** [sequence of ints, optional] Axes over which the DWT that created *coeffs* was performed. The default value of *None* corresponds to all axes.

### Returns

**coeff\_arr** [array-like] Wavelet transform coefficient array. All coefficients have been concatenated into a single array.

**coeff\_slices** [list] List of slices corresponding to each coefficient. As a 2D example, `coeff_arr[coeff_slices[1]['dd']]` would extract the first level detail coefficients from *coeff\_arr*.

**coeff\_shapes** [list] List of shapes corresponding to each coefficient. For example, in 2D, `coeff_shapes[1]['dd']` would contain the original shape of the first level detail coefficients array.

See also:

[\*unravel\\_coeffs\*](#) the inverse of *ravel\_coeffs*

## Examples

```
>>> import pywt
>>> cam = pywt.data.camera()
>>> coeffs = pywt.wavedecn(cam, wavelet='db2', level=3)
>>> arr, coeff_slices, coeff_shapes = pywt.ravel_coeffs(coeffs)
```

`pywt.unravel_coeffs` (*arr*, *coeff\_slices*, *coeff\_shapes*, *output\_format*='wavedecn')  
Unravel a raveled array of multilevel wavelet coefficients.

### Parameters

**arr** [array-like] An array containing all wavelet coefficients. This should have been generated by applying *ravel\_coeffs* to the output of *wavedec*, *wavedec2* or *wavedecn*.

**coeff\_slices** [list of tuples] List of slices corresponding to each coefficient as obtained from *ravel\_coeffs*.

**coeff\_shapes** [list of tuples] List of shapes corresponding to each coefficient as obtained from *ravel\_coeffs*.

**output\_format** [{‘wavedec’, ‘wavedec2’, ‘wavedecn’}, optional] Make the form of the unraveled coefficients compatible with this type of multilevel transform. The default is ‘wavedecn’.

### Returns

**coeffs**: list List of wavelet transform coefficients. The specific format of the list elements is determined by *output\_format*.

See also:

[\*ravel\\_coeffs\*](#) the inverse of *unravel\_coeffs*

## Examples

```
>>> import pywt
>>> from numpy.testing import assert_array_almost_equal
>>> cam = pywt.data.camera()
>>> coeffs = pywt.wavedecn(cam, wavelet='db2', level=3)
>>> arr, coeff_slices, coeff_shapes = pywt.ravel_coeffs(coeffs)
>>> coeffs_from_arr = pywt.unravel_coeffs(arr, coeff_slices, coeff_shapes)
```

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```
>>> cam_recon = pywt.waverecn(coeffs_from_arr, wavelet='db2')
>>> assert_array_almost_equal(cam, cam_recon)
```

### Multilevel: Total size of all coefficients - `wavedecn_size`

`pywt.wavedecn_size` (*shapes*)

Compute the total number of wavedecn coefficients.

#### Parameters

**shapes** [list of coefficient shapes] A set of coefficient shapes as returned by `wavedecn_shapes`. Alternatively, the user can specify a set of coefficients as returned by `wavedecn`.

#### Returns

**size** [int] The total number of coefficients.

### Examples

```
>>> import pywt
>>> data_shape = (64, 32)
>>> shapes = pywt.wavedecn_shapes(data_shape, 'db2', mode='periodization')
>>> pywt.wavedecn_size(shapes)
2048
>>> coeffs = pywt.wavedecn(np.ones(data_shape), 'sym4', mode='symmetric')
>>> pywt.wavedecn_size(coeffs)
3087
```

### Multilevel: n-d coefficient shapes - `wavedecn_shapes`

`pywt.wavedecn_shapes` (*shape, wavelet, mode='symmetric', level=None, axes=None*)

Subband shapes for a multilevel nD discrete wavelet transform.

#### Parameters

**shape** [sequence of ints] The shape of the data to be transformed.

**wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in *axes*.

**mode** [str or tuple of str, optional] Signal extension mode, see Modes (default: 'symmetric'). This can also be a tuple containing a mode to apply along each axis in *axes*.

**level** [int, optional] Decomposition level (must be  $\geq 0$ ). If level is None (default) then it will be calculated using the `dwt_max_level` function.

**axes** [sequence of ints, optional] Axes over which to compute the DWT. Axes may not be repeated. The default is None, which means transform all axes (`axes = range(data.ndim)`).

#### Returns

**shapes** [[cAn, {details\_level\_n}, ... {details\_level\_1}]] Coefficients shape list. Mirrors the output of `wavedecn`, except it contains only the shapes of the coefficient arrays rather than the arrays themselves.

## Examples

```
>>> import pywt
>>> pywt.wavedecn_shapes((64, 32), wavelet='db2', level=3, axes=(0, ))
[(10, 32), {'d': (10, 32)}, {'d': (18, 32)}, {'d': (33, 32)}]
```

## 5.2.9 Stationary Wavelet Transform

Stationary Wavelet Transform (SWT), also known as *Undecimated wavelet transform* or *Algorithme à trous* is a translation-invariance modification of the *Discrete Wavelet Transform* that does not decimate coefficients at every transformation level.

### Multilevel 1D `swt`

`pywt.swt` (*data*, *wavelet*, *level=None*, *start\_level=0*, *axis=-1*)  
Multilevel 1D stationary wavelet transform.

#### Parameters

- data** : Input signal
- wavelet** : Wavelet to use (Wavelet object or name)
- level** [int, optional] The number of decomposition steps to perform.
- start\_level** [int, optional] The level at which the decomposition will begin (it allows one to skip a given number of transform steps and compute coefficients starting from `start_level`) (default: 0)
- axis**: **int, optional** Axis over which to compute the SWT. If not given, the last axis is used.

#### Returns

**coeffs** [list] List of approximation and details coefficients pairs in order similar to `wavedec` function:

```
[(cAn, cDn), ..., (cA2, cD2), (cA1, cD1)]
```

where `n` equals input parameter `level`.

If `start_level = m` is given, then the beginning `m` steps are skipped:

```
[(cAm+n, cDm+n), ..., (cAm+1, cDm+1), (cAm, cDm)]
```

## Notes

The implementation here follows the “algorithm a-trous” and requires that the signal length along the transformed axis be a multiple of  $2^{**level}$ . If this is not the case, the user should pad up to an appropriate size using a function such as `numpy.pad`.

### Multilevel 2D `swt2`

`pywt.swt2` (*data*, *wavelet*, *level*, *start\_level=0*, *axes=(-2, -1)*)  
Multilevel 2D stationary wavelet transform.

**Parameters**

- data** [array\_like] 2D array with input data
- wavelet** [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple of wavelets to apply per axis in *axes*.
- level** [int] The number of decomposition steps to perform.
- start\_level** [int, optional] The level at which the decomposition will start (default: 0)
- axes** [2-tuple of ints, optional] Axes over which to compute the SWT. Repeated elements are not allowed.

**Returns**

- coeffs** [list] Approximation and details coefficients (for `start_level = m`):

```
[
    (cA_m+level,
     (cH_m+level, cV_m+level, cD_m+level)
    ),
    ...,
    (cA_m+1,
     (cH_m+1, cV_m+1, cD_m+1)
    ),
    (cA_m,
     (cH_m, cV_m, cD_m)
    )
]
```

where *cA* is approximation, *cH* is horizontal details, *cV* is vertical details, *cD* is diagonal details and *m* is `start_level`.

**Notes**

The implementation here follows the “algorithm a-trous” and requires that the signal length along the transformed axes be a multiple of  $2^{**level}$ . If this is not the case, the user should pad up to an appropriate size using a function such as `numpy.pad`.

**Multilevel n-dimensional `swt_n`**

`pywt.swt_n` (*data*, *wavelet*, *level*, *start\_level=0*, *axes=None*)  
n-dimensional stationary wavelet transform.

**Parameters**

- data** [array\_like] n-dimensional array with input data.
- wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple of wavelets to apply per axis in *axes*.
- level** [int] The number of decomposition steps to perform.
- start\_level** [int, optional] The level at which the decomposition will start (default: 0)
- axes** [sequence of ints, optional] Axes over which to compute the SWT. A value of `None` (the default) selects all axes. Axes may not be repeated.

**Returns**

[{**coeffs\_level\_n**}, ..., {**coeffs\_level\_1**}]: list of dict Results for each level are arranged in a dictionary, where the key specifies the transform type on each dimension and value is a n-dimensional coefficients array.

For example, for a 2D case the result at a given level will look something like this:

```
{'aa': <coeffs> # A(LL) - approx. on 1st dim, approx. on 2nd dim
'ad': <coeffs> # V(LH) - approx. on 1st dim, det. on 2nd dim
'da': <coeffs> # H(HL) - det. on 1st dim, approx. on 2nd dim
'dd': <coeffs> # D(HH) - det. on 1st dim, det. on 2nd dim
}
```

For user-specified `axes`, the order of the characters in the dictionary keys map to the specified `axes`.

## Notes

The implementation here follows the “algorithm a-trous” and requires that the signal length along the transformed axes be a multiple of `2**level`. If this is not the case, the user should pad up to an appropriate size using a function such as `numpy.pad`.

## Maximum decomposition level - `swt_max_level`

`pywt.swt_max_level(input_len)`

Calculates the maximum level of Stationary Wavelet Transform for data of given length.

### Parameters

**input\_len** [int] Input data length.

### Returns

**max\_level** [int] Maximum level of Stationary Wavelet Transform for data of given length.

## Notes

For the current implementation of the stationary wavelet transform, this corresponds to the number of times `input_len` is evenly divisible by two. In other words, for an n-level transform, the signal length must be a multiple of `2**n`. `numpy.pad` can be used to pad a signal up to an appropriate length as needed.

## 5.2.10 Inverse Stationary Wavelet Transform

Inverse *stationary wavelet transforms* are provided.

**Note:** These inverse transforms are not yet optimized for speed. Only, the n-dimensional inverse transform currently has `axes` support.

### Multilevel 1D `iswt`

`pywt.iswt(coeffs, wavelet)`

Multilevel 1D inverse discrete stationary wavelet transform.

### Parameters

**coeffs** [array\_like] Coefficients list of tuples:

```
[(cAn, cDn), ..., (cA2, cD2), (cA1, cD1)]
```

where cA is approximation, cD is details. Index 1 corresponds to `start_level` from `pywt.swt`.

**wavelet** [Wavelet object or name string] Wavelet to use

**Returns**

**1D array of reconstructed data.**

**Examples**

```
>>> import pywt
>>> coeffs = pywt.swt([1,2,3,4,5,6,7,8], 'db2', level=2)
>>> pywt.iswt(coeffs, 'db2')
array([ 1.,  2.,  3.,  4.,  5.,  6.,  7.,  8.]
```

**Multilevel 2D iswt2**

`pywt.iswt2(coeffs, wavelet)`

Multilevel 2D inverse discrete stationary wavelet transform.

**Parameters**

**coeffs** [list] Approximation and details coefficients:

```
[
    (cA_n,
     (cH_n, cV_n, cD_n)
    ),
    ...,
    (cA_2,
     (cH_2, cV_2, cD_2)
    ),
    (cA_1,
     (cH_1, cV_1, cD_1)
    )
]
```

where cA is approximation, cH is horizontal details, cV is vertical details, cD is diagonal details and n is the number of levels. Index 1 corresponds to `start_level` from `pywt.swt2`.

**wavelet** [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a 2-tuple of wavelets to apply per axis.

**Returns**

**2D array of reconstructed data.**

## Examples

```
>>> import pywt
>>> coeffs = pywt.swt2([[1,2,3,4],[5,6,7,8],
...                    [9,10,11,12],[13,14,15,16]],
...                    'db1', level=2)
>>> pywt.iswt2(coeffs, 'db1')
array([[ 1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.],
       [ 9., 10., 11., 12.],
       [13., 14., 15., 16.]])
```

### Multilevel n-dimensional `iswtn`

`pywt.iswtn` (*coeffs*, *wavelet*, *axes=None*)

Multilevel nD inverse discrete stationary wavelet transform.

#### Parameters

**coeffs** [list] [{coeffs\_level\_n}, ..., {coeffs\_level\_1}]: list of dict

**wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple of wavelets to apply per axis in *axes*.

**axes** [sequence of ints, optional] Axes over which to compute the inverse SWT. Axes may not be repeated. The default is `None`, which means transform all axes (`axes = range(data.ndim)`).

#### Returns

**nD array of reconstructed data.**

## Examples

```
>>> import pywt
>>> coeffs = pywt.swtn([[1,2,3,4],[5,6,7,8],
...                    [9,10,11,12],[13,14,15,16]],
...                    'db1', level=2)
>>> pywt.iswtn(coeffs, 'db1')
array([[ 1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.],
       [ 9., 10., 11., 12.],
       [13., 14., 15., 16.]])
```

## 5.2.11 Wavelet Packets

New in version 0.2.

Version 0.2 of PyWavelets includes many new features and improvements. One of such new feature is a two-dimensional wavelet packet transform structure that is almost completely sharing programming interface with the one-dimensional tree structure.

In order to achieve this simplification, a new inheritance scheme was used in which a *BaseNode* base node class is a superclass for both *Node* and *Node2D* node classes.

The node classes are used as data wrappers and can be organized in trees (binary trees for 1D transform case and quad-trees for the 2D one). They are also superclasses to the *WaveletPacket* class and *WaveletPacket2D* class that are used as the decomposition tree roots and contain a couple additional methods.

The below diagram illustrates the inheritance tree:

- *BaseNode* - common interface for 1D and 2D nodes:
  - *Node* - data carrier node in a 1D decomposition tree
    - \* *WaveletPacket* - 1D decomposition tree root node
  - *Node2D* - data carrier node in a 2D decomposition tree
    - \* *WaveletPacket2D* - 2D decomposition tree root node

### BaseNode - a common interface of WaveletPacket and WaveletPacket2D

```
class pywt.BaseNode
class pywt.Node(BaseNode)
class pywt.WaveletPacket(Node)
class pywt.Node2D(BaseNode)
class pywt.WaveletPacket2D(Node2D)
```

---

**Note:** The *BaseNode* is a base class for *Node* and *Node2D*. It should not be used directly unless creating a new transformation type. It is included here to document the common interface of 1D and 2D node and wavelet packet transform classes.

---

`__init__` (*parent*, *data*, *node\_name*)

#### Parameters

- **parent** – parent node. If parent is *None* then the node is considered detached.
- **data** – data associated with the node. 1D or 2D numeric array, depending on the transform type.
- **node\_name** – a name identifying the coefficients type. See *Node.node\_name* and *Node2D.node\_name* for information on the accepted subnodes names.

#### data

Data associated with the node. 1D or 2D numeric array (depends on the transform type).

#### parent

Parent node. Used in tree navigation. *None* for root node.

#### wavelet

*Wavelet* used for decomposition and reconstruction. Inherited from parent node.

#### mode

Signal extension *mode* for the *dwt()* (*dwt2()*) and *idwt()* (*idwt2()*) decomposition and reconstruction functions. Inherited from parent node.

#### level

Decomposition level of the current node. 0 for root (original data), 1 for the first decomposition level, etc.

#### path

Path string defining position of the node in the decomposition tree.

**node\_name**

Node name describing data coefficients type of the current subnode.

See `Node.node_name` and `Node2D.node_name`.

**maxlevel**

Maximum allowed level of decomposition. Evaluated from parent or child nodes.

**is\_empty**

Checks if data attribute is None.

**has\_any\_subnode**

Checks if node has any subnodes (is not a leaf node).

**decompose()**

Performs Discrete Wavelet Transform on the data and returns transform coefficients.

**reconstruct** (`[update=False]`)

Performs Inverse Discrete Wavelet Transform on subnodes coefficients and returns reconstructed data for the current level.

**Parameters** `update` – If set, the data attribute will be updated with the reconstructed value.

---

**Note:** Descends to subnodes and recursively calls `reconstruct()` on them.

---

**get\_subnode** (`part[, decompose=True]`)

Returns subnode or None (see *decomposition* flag description).

**Parameters**

- **part** – Subnode name
- **decompose** – If True and subnode does not exist, it will be created using coefficients from the DWT decomposition of the current node.

**\_\_getitem\_\_** (`path`)

Used to access nodes in the decomposition tree by string path.

**Parameters** `path` – Path string composed from valid node names. See `Node.node_name` and `Node2D.node_name` for node naming convention.

Similar to `get_subnode()` method with `decompose=True`, but can access nodes on any level in the decomposition tree.

If node does not exist yet, it will be created by decomposition of its parent node.

**\_\_setitem\_\_** (`path, data`)

Used to set node or node's data in the decomposition tree. Nodes are identified by string path.

**Parameters**

- **path** – Path string composed from valid node names. See `Node.node_name` and `Node2D.node_name` for node naming convention.
- **data** – numeric array or `BaseNode` subclass.

**\_\_delitem\_\_** (`path`)

Used to delete node from the decomposition tree.

**Parameters** `path` – Path string composed from valid node names. See `Node.node_name` and `Node2D.node_name` for node naming convention.

`get_leaf_nodes` (`[decompose=False]`)

Traverses through the decomposition tree and collects leaf nodes (nodes without any subnodes).

**Parameters** `decompose` – If `decompose` is `True`, the method will try to decompose the tree up to the maximum level.

`walk` (`self, func`, [`args=()`], [`kwargs={}`], [`decompose=True` ] ] )

Traverses the decomposition tree and calls `func` (`node`, `*args`, `**kwargs`) on every node. If `func` returns `True`, descending to subnodes will continue.

**Parameters**

- **func** – callable accepting `BaseNode` as the first param and optional positional and keyword arguments:

```
func (node, *args, **kwargs)
```

- **decompose** – If `decompose` is `True` (default), the method will also try to decompose the tree up to the maximum level.

**Args** arguments to pass to the `func`

**Kwargs** keyword arguments to pass to the `func`

`walk_depth` (`self, func`, [`args=()`], [`kwargs={}`], [`decompose=False` ] ] )

Similar to `walk` () but traverses the tree in depth-first order.

**Parameters**

- **func** – callable accepting `BaseNode` as the first param and optional positional and keyword arguments:

```
func (node, *args, **kwargs)
```

- **decompose** – If `decompose` is `True`, the method will also try to decompose the tree up to the maximum level.

**Args** arguments to pass to the `func`

**Kwargs** keyword arguments to pass to the `func`

## WaveletPacket and WaveletPacket tree Node

```
class pywt.Node (BaseNode)
```

```
class pywt.WaveletPacket (Node)
```

**node\_name**

Node name describing data coefficients type of the current subnode.

For `WaveletPacket` case it is just as in `dwt` ():

- a - approximation coefficients
- d - details coefficients

**decompose** ()

See also:

- `dwt` () for 1D Discrete Wavelet Transform output coefficients.

```
class pywt.WaveletPacket (Node)
```

```
__init__ (data, wavelet[, mode='symmetric'[, maxlevel=None ]])
```

#### Parameters

- **data** – data associated with the node. 1D numeric array.
- **wavelet** – Wavelet to use in the transform. This can be a name of the wavelet from the `wavelist()` list or a `Wavelet` object instance.
- **mode** – Signal extension *mode* for the `dwt()` and `idwt()` decomposition and reconstruction functions.
- **maxlevel** – Maximum allowed level of decomposition. If not specified it will be calculated based on the wavelet and data length using `pywt.dwt_max_level()`.

```
get_level (level[, order="natural"[, decompose=True ]])
```

Collects nodes from the given level of decomposition.

#### Parameters

- **level** – Specifies decomposition `level` from which the nodes will be collected.
- **order** – Specifies nodes order - natural (`natural`) or frequency (`freq`).
- **decompose** – If set then the method will try to decompose the data up to the specified `level`.

If nodes at the given level are missing (i.e. the tree is partially decomposed) and the `decompose` is set to `False`, only existing nodes will be returned.

## WaveletPacket2D and WaveletPacket2D tree Node2D

```
class pywt.Node2D (BaseNode)
```

```
class pywt.WaveletPacket2D (Node2D)
```

`node_name`

For `WaveletPacket2D` case it is just as in `dwt2()`:

- a - approximation coefficients (*LL*)
- h - horizontal detail coefficients (*LH*)
- v - vertical detail coefficients (*HL*)
- d - diagonal detail coefficients (*HH*)

`decompose()`

See also:

`dwt2()` for 2D Discrete Wavelet Transform output coefficients.

```
expand_2d_path(self, path):
```

```
class pywt.WaveletPacket2D (Node2D)
```

```
__init__ (data, wavelet[, mode='symmetric'[, maxlevel=None ]])
```

#### Parameters

- **data** – data associated with the node. 2D numeric array.
- **wavelet** – Wavelet to use in the transform. This can be a name of the wavelet from the `wavelist()` list or a `Wavelet` object instance.
- **mode** – Signal extension *mode* for the `dwt()` and `idwt()` decomposition and reconstruction functions.
- **maxlevel** – Maximum allowed level of decomposition. If not specified it will be calculated based on the `wavelet` and `data` length using `pywt.dwt_max_level()`.

`get_level(level[, order="natural", decompose=True])`

Collects nodes from the given level of decomposition.

#### Parameters

- **level** – Specifies decomposition `level` from which the nodes will be collected.
- **order** – Specifies nodes order - `natural` (`natural`) or `frequency` (`freq`).
- **decompose** – If set then the method will try to decompose the data up to the specified `level`.

If nodes at the given level are missing (i.e. the tree is partially decomposed) and the `decompose` is set to `False`, only existing nodes will be returned.

## 5.2.12 Continuous Wavelet Transform (CWT)

This section describes functions used to perform single continuous wavelet transforms.

### Single level - `cwt`

`pywt.cwt(data, scales, wavelet)`

One dimensional Continuous Wavelet Transform.

#### Parameters

**data** [array\_like] Input signal

**scales** [array\_like] The wavelet scales to use. One can use `f = scale2frequency(scale, wavelet)/sampling_period` to determine what physical frequency, `f`. Here, `f` is in hertz when the `sampling_period` is given in seconds.

**wavelet** [Wavelet object or name] Wavelet to use

**sampling\_period** [float] Sampling period for the frequencies output (optional). The values computed for `coefs` are independent of the choice of `sampling_period` (i.e. `scales` is not scaled by the sampling period).

#### Returns

**coefs** [array\_like] Continuous wavelet transform of the input signal for the given scales and wavelet

**frequencies** [array\_like] If the unit of sampling period are seconds and given, than frequencies are in hertz. Otherwise, a sampling period of 1 is assumed.

## Notes

Size of coefficients arrays depends on the length of the input array and the length of given scales.

## Examples

```
>>> import pywt
>>> import numpy as np
>>> import matplotlib.pyplot as plt
>>> x = np.arange(512)
>>> y = np.sin(2*np.pi*x/32)
>>> coef, freqs=pywt.cwt(y,np.arange(1,129),'gaus1')
>>> plt.matshow(coef)
>>> plt.show()
-----
>>> import pywt
>>> import numpy as np
>>> import matplotlib.pyplot as plt
>>> t = np.linspace(-1, 1, 200, endpoint=False)
>>> sig = np.cos(2 * np.pi * 7 * t) + np.real(np.exp(-7*(t-0.4)**2)*np.
↳exp(1j*2*np.pi*2*(t-0.4)))
>>> widths = np.arange(1, 31)
>>> cwtmatr, freqs = pywt.cwt(sig, widths, 'mexh')
>>> plt.imshow(cwtmatr, extent=[-1, 1, 1, 31], cmap='PRGn', aspect='auto',
...           vmax=abs(cwtmatr).max(), vmin=-abs(cwtmatr).max())
>>> plt.show()
```

## Continuous Wavelet Families

A variety of continuous wavelets have been implemented. A list of the available wavelet names compatible with `cwt` can be obtained by:

```
wavlist = pywt.wavelist(kind='continuous')
```

## Mexican Hat Wavelet

The mexican hat wavelet "mexh" is given by:

$$\psi(t) = \frac{2}{\sqrt{3}\sqrt[4]{\pi}} \exp^{-\frac{t^2}{2}} (1 - t^2)$$

where the constant out front is a normalization factor so that the wavelet has unit energy.

## Morlet Wavelet

The Morlet wavelet "morl" is given by:

$$\psi(t) = \exp^{-\frac{t^2}{2}} \cos(5t)$$

## Complex Morlet Wavelets

The complex Morlet wavelet ("cmorB-C" with floating point values B, C) is given by:

$$\psi(t) = \frac{1}{\sqrt{\pi B}} \exp^{-\frac{t^2}{B}} \exp^{j2\pi Ct}$$

where  $B$  is the bandwidth and  $C$  is the center frequency.

## Gaussian Derivative Wavelets

The Gaussian wavelets ("gausP" where P is an integer between 1 and 8) correspond to the Pth order derivatives of the function:

$$\psi(t) = C \exp^{-t^2}$$

where  $C$  is an order-dependent normalization constant.

## Complex Gaussian Derivative Wavelets

The complex Gaussian wavelets ("cgauP" where P is an integer between 1 and 8) correspond to the Pth order derivatives of the function:

$$\psi(t) = C \exp^{-jt} \exp^{-t^2}$$

where  $C$  is an order-dependent normalization constant.

## Shannon Wavelets

The Shannon wavelets ("shanB-C" with floating point values B and C) correspond to the following wavelets:

$$\psi(t) = \sqrt{B} \frac{\sin(\pi B t)}{\pi B t} \exp^{j2\pi C t}$$

where  $B$  is the bandwidth and  $C$  is the center frequency.

## Frequency B-Spline Wavelets

The frequency B-spline wavelets ("fpspM-B-C" with integer M and floating point B, C) correspond to the following wavelets:

$$\psi(t) = \sqrt{B} \left[ \frac{\sin(\pi B \frac{t}{M})}{\pi B \frac{t}{M}} \right]^M \exp^{2j\pi C t}$$

where  $M$  is the spline order,  $B$  is the bandwidth and  $C$  is the center frequency.

## Choosing the scales for cwt

For each of the wavelets described below, the implementation in PyWavelets evaluates the wavelet function for  $t$  over the range `[wavelet.lower_bound, wavelet.upper_bound]` (with default range `[-8, 8]`). `scale = 1` corresponds to the case where the extent of the wavelet is `(wavelet.upper_bound - wavelet.lower_bound + 1)` samples of the digital signal being analyzed. Larger scales correspond to stretching of the wavelet. For example, at `scale=10` the wavelet is stretched by a factor of 10, making it sensitive to lower frequencies in the signal.

To relate a given scale to a specific signal frequency, the sampling period of the signal must be known. `pywt.scale2frequency()` can be used to convert a list of scales to their corresponding frequencies. The proper choice of scales depends on the chosen wavelet, so `pywt.scale2frequency()` should be used to get an idea of an appropriate range for the signal of interest.

For the `cmor`, `fbasp` and `shan` wavelets, the user can specify a specific a normalized center frequency. A value of 1.0 corresponds to  $1/dt$  where  $dt$  is the sampling period. In other words, when analyzing a signal sampled at 100 Hz, a center frequency of 1.0 corresponds to  $\sim 100$  Hz at `scale = 1`. This is above the Nyquist rate of 50 Hz, so for this particular wavelet, one would analyze a signal using scales `>= 2`.

```
>>> import numpy as np
>>> import pywt
>>> dt = 0.01 # 100 Hz sampling
>>> frequencies = pywt.scale2frequency('cmor1.5-1.0', [1, 2, 3, 4]) / dt
>>> frequencies
array([ 100.          ,  50.          ,  33.33333333,  25.          ])
```

The CWT in PyWavelets is applied to discrete data by convolution with samples of the integral of the wavelet. If `scale` is too low, this will result in a discrete filter that is inadequately sampled leading to aliasing as shown in the example below. Here the wavelet is `'cmor1.5-1.0'`. The left column of the figure shows the discrete filters used in the convolution at various scales. The right column are the corresponding Fourier power spectra of each filter. For scales 1 and 2 it can be seen that aliasing due to violation of the Nyquist limit occurs.

```
import numpy as np
import pywt
import matplotlib.pyplot as plt

wav = pywt.ContinuousWavelet('cmor1.5-1.0')

# print the range over which the wavelet will be evaluated
print("Continuous wavelet will be evaluated over the range [{}, {}]".format(
    wav.lower_bound, wav.upper_bound))

width = wav.upper_bound - wav.lower_bound

scales = [1, 2, 3, 4, 10, 15]

max_len = int(np.max(scales)*width + 1)
t = np.arange(max_len)
fig, axes = plt.subplots(len(scales), 2, figsize=(12, 6))
for n, scale in enumerate(scales):

    # The following code is adapted from the internals of cwt
    int_psi, x = pywt.integrate_wavelet(wav, precision=10)
    step = x[1] - x[0]
    j = np.floor(
        np.arange(scale * width + 1) / (scale * step))
    if np.max(j) >= np.size(int_psi):
```

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```

    j = np.delete(j, np.where((j >= np.size(int_psi)))[0])
    j = j.astype(np.int)

    # normalize int_psi for easier plotting
    int_psi /= np.abs(int_psi).max()

    # discrete samples of the integrated wavelet
    filt = int_psi[j][::-1]

    # The CWT consists of convolution of filt with the signal at this scale
    # Here we plot this discrete convolution kernel at each scale.

    nt = len(filt)
    t = np.linspace(-nt//2, nt//2, nt)
    axes[n, 0].plot(t, filt.real, t, filt.imag)
    axes[n, 0].set_xlim([-max_len//2, max_len//2])
    axes[n, 0].set_ylim([-1, 1])
    axes[n, 0].text(50, 0.35, 'scale = {}'.format(scale))

    f = np.linspace(-np.pi, np.pi, max_len)
    filt_fft = np.fft.fftshift(np.fft.fft(filt, n=max_len))
    filt_fft /= np.abs(filt_fft).max()
    axes[n, 1].plot(f, np.abs(filt_fft)**2)
    axes[n, 1].set_xlim([-np.pi, np.pi])
    axes[n, 1].set_ylim([0, 1])
    axes[n, 1].set_xticks([-np.pi, 0, np.pi])
    axes[n, 1].set_xticklabels([r'$-\pi$', '0', r'$\pi$'])
    axes[n, 1].grid(True, axis='x')
    axes[n, 1].text(np.pi/2, 0.5, 'scale = {}'.format(scale))

axes[n, 0].set_xlabel('time (samples)')
axes[n, 1].set_xlabel('frequency (radians)')
axes[0, 0].legend(['real', 'imaginary'], loc='upper left')
axes[0, 1].legend(['Power'], loc='upper left')
axes[0, 0].set_title('filter')
axes[0, 1].set_title(r'|FFT(filter)|$^2$')

```

### 5.2.13 Thresholding functions

The thresholding helper module implements the most popular signal thresholding functions.

#### Thresholding

`pywt.threshold(data, value, mode='soft', substitute=0)`

Thresholds the input data depending on the mode argument.

In *soft* thresholding [1], data values with absolute value less than *param* are replaced with *substitute*. Data values with absolute value greater or equal to the thresholding value are shrunk toward zero by *value*. In other words, the new value is  $\text{data}/\text{np.abs}(\text{data}) * \text{np.maximum}(\text{np.abs}(\text{data}) - \text{value}, 0)$ .

In *hard* thresholding, the data values where their absolute value is less than the value *param* are replaced with *substitute*. Data values with absolute value greater or equal to the thresholding value stay untouched.

*garotte* corresponds to the Non-negative garrote threshold [2], [3]. It is intermediate between *hard* and *soft* thresholding. It behaves like *soft* thresholding for small data values and approaches *hard* thresholding for

large data values.

In `greater` thresholding, the data is replaced with *substitute* where data is below the thresholding value. Greater data values pass untouched.

In `less` thresholding, the data is replaced with *substitute* where data is above the thresholding value. Lesser data values pass untouched.

Both `hard` and `soft` thresholding also support complex-valued data.

### Parameters

**data** [array\_like] Numeric data.

**value** [scalar] Thresholding value.

**mode** [{'soft', 'hard', 'greater', 'less'}] Decides the type of thresholding to be applied on input data. Default is 'soft'.

**substitute** [float, optional] Substitute value (default: 0).

### Returns

**output** [array] Thresholded array.

See also:

`threshold_firm`

### References

[1], [2], [3]

### Examples

```
>>> import numpy as np
>>> import pywt
>>> data = np.linspace(1, 4, 7)
>>> data
array([ 1. ,  1.5,  2. ,  2.5,  3. ,  3.5,  4. ])
>>> pywt.threshold(data, 2, 'soft')
array([ 0. ,  0. ,  0. ,  0.5,  1. ,  1.5,  2. ])
>>> pywt.threshold(data, 2, 'hard')
array([ 0. ,  0. ,  2. ,  2.5,  3. ,  3.5,  4. ])
>>> pywt.threshold(data, 2, 'garotte')
array([ 0.          ,  0.          ,  0.          ,  0.9          ,  1.66666667,
        2.35714286,  3.          ])
>>> pywt.threshold(data, 2, 'greater')
array([ 0. ,  0. ,  2. ,  2.5,  3. ,  3.5,  4. ])
>>> pywt.threshold(data, 2, 'less')
array([ 1. ,  1.5,  2. ,  0. ,  0. ,  0. ,  0. ])
```

`pywt.threshold_firm`(*data*, *value\_low*, *value\_high*)

Firm threshold.

The approach is intermediate between soft and hard thresholding [1]. It behaves the same as soft-thresholding for values below *value\_low* and the same as hard-thresholding for values above *thresh\_high*. For intermediate values, the thresholded value is in between that corresponding to soft or hard thresholding.

### Parameters

**data** [array-like] The data to threshold. This can be either real or complex-valued.

**value\_low** [float] Any values smaller than *value\_low* will be set to zero.

**value\_high** [float] Any values larger than *value\_high* will not be modified.

### Returns

**val\_new** [array-like] The values after firm thresholding at the specified thresholds.

### See also:

*threshold*

### Notes

This thresholding technique is also known as semi-soft thresholding [2].

For each value, *x*, in *data*. This function computes:

```
if np.abs(x) <= value_low:
    return 0
elif np.abs(x) > value_high:
    return x
elif value_low < np.abs(x) and np.abs(x) <= value_high:
    return x * value_high * (1 - value_low/x) / (value_high - value_low)
```

*firm* is a continuous function (like soft thresholding), but is unbiased for large values (like hard thresholding).

If *value\_high* == *value\_low* this function becomes hard-thresholding. If *value\_high* is infinity, this function becomes soft-thresholding.

### References

[1],[2]

The left panel of the figure below illustrates that non-negative Garotte thresholding is intermediate between soft and hard thresholding. Firm thresholding transitions between soft and hard thresholding behavior. It requires a pair of threshold values that define the width of the transition region.

```
import numpy as np
import matplotlib.pyplot as plt
import pywt

s = np.linspace(-4, 4, 1000)

s_soft = pywt.threshold(s, value=0.5, mode='soft')
s_hard = pywt.threshold(s, value=0.5, mode='hard')
s_garotte = pywt.threshold(s, value=0.5, mode='garotte')
s_firm1 = pywt.threshold_firm(s, value_low=0.5, value_high=1)
s_firm2 = pywt.threshold_firm(s, value_low=0.5, value_high=2)
s_firm3 = pywt.threshold_firm(s, value_low=0.5, value_high=4)

fig, ax = plt.subplots(1, 2, figsize=(10, 4))
ax[0].plot(s, s_soft)
ax[0].plot(s, s_hard)
ax[0].plot(s, s_garotte)
ax[0].legend(['soft (0.5)', 'hard (0.5)', 'non-neg. garotte (0.5)'])
```

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```

ax[0].set_xlabel('input value')
ax[0].set_ylabel('thresholded value')

ax[1].plot(s, s_soft)
ax[1].plot(s, s_hard)
ax[1].plot(s, s_firm1)
ax[1].plot(s, s_firm2)
ax[1].plot(s, s_firm3)
ax[1].legend(['soft (0.5)', 'hard (0.5)', 'firm(0.5, 1)', 'firm(0.5, 2)',
            'firm(0.5, 4)'])
ax[1].set_xlabel('input value')
ax[1].set_ylabel('thresholded value')
plt.show()

```

## 5.2.14 Other functions

### Integrating wavelet functions

`pywt.integrate_wavelet` (*wavelet*, *precision*=8)

Integrate *psi* wavelet function from  $-\infty$  to *x* using the rectangle integration method.

#### Parameters

**wavelet** [Wavelet instance or str] Wavelet to integrate. If a string, should be the name of a wavelet.

**precision** [int, optional] Precision that will be used for wavelet function approximation computed with the `wavefun(level=precision)` Wavelet's method (default: 8).

#### Returns

[**int\_psi**, **x**] : for orthogonal wavelets

[**int\_psi\_d**, **int\_psi\_r**, **x**] : for other wavelets

### Examples

```

>>> from pywt import Wavelet, integrate_wavelet
>>> wavelet1 = Wavelet('db2')
>>> [int_psi, x] = integrate_wavelet(wavelet1, precision=5)
>>> wavelet2 = Wavelet('bior1.3')
>>> [int_psi_d, int_psi_r, x] = integrate_wavelet(wavelet2, precision=5)

```

The result of the call depends on the `wavelet` argument:

- for orthogonal and continuous wavelets - an integral of the wavelet function specified on an x-grid:

```
[int_psi, x_grid] = integrate_wavelet(wavelet, precision)
```

- for other wavelets - integrals of decomposition and reconstruction wavelet functions and a corresponding x-grid:

```
[int_psi_d, int_psi_r, x_grid] = integrate_wavelet(wavelet, precision)
```

## Central frequency of $\psi$ wavelet function

`pywt.central_frequency` (*wavelet*, *precision=8*)

Computes the central frequency of the  $\psi$  wavelet function.

### Parameters

**wavelet** [Wavelet instance, str or tuple] Wavelet to integrate. If a string, should be the name of a wavelet.

**precision** [int, optional] Precision that will be used for wavelet function approximation computed with the `wavefun(level=precision)` Wavelet's method (default: 8).

### Returns

scalar

`pywt.scale2frequency` (*wavelet*, *scale*, *precision=8*)

### Parameters

**wavelet** [Wavelet instance or str] Wavelet to integrate. If a string, should be the name of a wavelet.

**scale** [scalar]

**precision** [int, optional] Precision that will be used for wavelet function approximation computed with `wavelet.wavefun(level=precision)`. Default is 8.

### Returns

**freq** [scalar]

## Quadrature Mirror Filter

`pywt.qmf` (*filt*)

Returns the Quadrature Mirror Filter(QMF).

The magnitude response of QMF is mirror image about  $\pi/2$  of that of the input filter.

### Parameters

**filt** [array\_like] Input filter for which QMF needs to be computed.

### Returns

**qm\_filter** [ndarray] Quadrature mirror of the input filter.

## Orthogonal Filter Banks

`pywt.orthogonal_filter_bank` (*scaling\_filter*)

Returns the orthogonal filter bank.

The orthogonal filter bank consists of the HPFs and LPFs at decomposition and reconstruction stage for the input scaling filter.

### Parameters

**scaling\_filter** [array\_like] Input scaling filter (father wavelet).

### Returns

**orth\_filt\_bank** [tuple of 4 ndarrays] The orthogonal filter bank of the input scaling filter in the order : 1] Decomposition LPF 2] Decomposition HPF 3] Reconstruction LPF 4] Reconstruction HPF

## Example Datasets

The following example datasets are available in the module `pywt.data`:

name	description
ecg	ECG waveform (1024 samples)
aero	grayscale image (512x512)
ascent	grayscale image (512x512)
camera	grayscale image (512x512)
nino	sea surface temperature (264 samples)
demo_signal	various synthetic 1d test signals

Each can be loaded via a function of the same name.

`pywt.data.demo_signal` (*name*='Bumps', *n*=None)  
Simple 1D wavelet test functions.

This function can generate a number of common 1D test signals used in papers by David Donoho and colleagues (e.g. [1]) as well as the wavelet book by Stéphane Mallat [2].

### Parameters

**name** [{ 'Blocks', 'Bumps', 'HeaviSine', 'Doppler', ... }] The type of test signal to generate (*name* is case-insensitive). If *name* is set to *'list'*, a list of the available test functions is returned.

**n** [int or None] The length of the test signal. This should be provided for all test signals except *'Gabor'* and *'sineoneoverx'* which have a fixed length.

### Returns

**f** [np.ndarray] Array of length *n* corresponding to the specified test signal type.

## Notes

This function is a partial reimplementation of the *MakeSignal* function from the [Wavelab](<https://statweb.stanford.edu/~wavelab/>) toolbox. These test signals are provided with permission of Dr. Donoho to encourage reproducible research.

## References

[1], [2]

### Example:

```
>>> import pywt
>>> camera = pywt.data.camera()
>>> doppler = pywt.data.demo_signal('doppler')
>>> available_signals = pywt.data.demo_signal('list')
```

## 5.3 Usage examples

The following examples are used as doctest regression tests written using reST markup. They are included in the documentation since they contain various useful examples illustrating how to use and how not to use PyWavelets.

For more usage examples see the `demo` directory in the source package.

### 5.3.1 The Wavelet object

#### Wavelet families and builtin Wavelets names

*Wavelet* objects are really a handy carriers of a bunch of DWT-specific data like *quadrature mirror filters* and some general properties associated with them.

At first let's go through the methods of creating a *Wavelet* object. The easiest and the most convenient way is to use builtin named Wavelets.

These wavelets are organized into groups called wavelet families. The most commonly used families are:

```
>>> import pywt
>>> pywt.families()
['haar', 'db', 'sym', 'coif', 'bior', 'rbio', 'dmey', 'gaus', 'mexh', 'morl', 'cgau',
 ↪ 'shan', 'fbsp', 'cmor']
```

The `wavelist()` function with family name passed as an argument is used to obtain the list of wavelet names in each family.

```
>>> for family in pywt.families():
...     print("%s family: " % family + ', '.join(pywt.wavelist(family)))
haar family: haar
db family: db1, db2, db3, db4, db5, db6, db7, db8, db9, db10, db11, db12, db13, db14, ↪
↪ db15, db16, db17, db18, db19, db20, db21, db22, db23, db24, db25, db26, db27, db28, ↪
↪ db29, db30, db31, db32, db33, db34, db35, db36, db37, db38
sym family: sym2, sym3, sym4, sym5, sym6, sym7, sym8, sym9, sym10, sym11, sym12, ↪
↪ sym13, sym14, sym15, sym16, sym17, sym18, sym19, sym20
coif family: coif1, coif2, coif3, coif4, coif5, coif6, coif7, coif8, coif9, coif10, ↪
↪ coif11, coif12, coif13, coif14, coif15, coif16, coif17
bior family: bior1.1, bior1.3, bior1.5, bior2.2, bior2.4, bior2.6, bior2.8, bior3.1, ↪
↪ bior3.3, bior3.5, bior3.7, bior3.9, bior4.4, bior5.5, bior6.8
rbio family: rbio1.1, rbio1.3, rbio1.5, rbio2.2, rbio2.4, rbio2.6, rbio2.8, rbio3.1, ↪
↪ rbio3.3, rbio3.5, rbio3.7, rbio3.9, rbio4.4, rbio5.5, rbio6.8
dmey family: dmey
gaus family: gaus1, gaus2, gaus3, gaus4, gaus5, gaus6, gaus7, gaus8
mexh family: mexh
morl family: morl
cgau family: cgau1, cgau2, cgau3, cgau4, cgau5, cgau6, cgau7, cgau8
shan family: shan
fbsp family: fbsp
cmor family: cmor
```

To get the full list of builtin wavelets' names just use the `wavelist()` with no argument.

#### Creating Wavelet objects

Now when we know all the names let's finally create a *Wavelet* object:

```
>>> w = pywt.Wavelet('db3')
```

So.. that's it.

## Wavelet properties

But what can we do with *Wavelet* objects? Well, they carry some interesting information.

First, let's try printing a *Wavelet* object. This shows a brief information about its name, its family name and some properties like orthogonality and symmetry.

```
>>> print(w)
Wavelet db3
  Family name:   Daubechies
  Short name:    db
  Filters length: 6
  Orthogonal:    True
  Biorthogonal:  True
  Symmetry:      asymmetric
  DWT:           True
  CWT:           False
```

But the most important information are the wavelet filters coefficients, which are used in *Discrete Wavelet Transform*. These coefficients can be obtained via the *dec\_lo*, *Wavelet.dec\_hi*, *rec\_lo* and *rec\_hi* attributes, which corresponds to lowpass and highpass decomposition filters and lowpass and highpass reconstruction filters respectively:

```
>>> def print_array(arr):
...     print("[%s]" % ", ".join(["%.14f" % x for x in arr]))
```

Another way to get the filters data is to use the *filter\_bank* attribute, which returns all four filters in a tuple:

```
>>> w.filter_bank == (w.dec_lo, w.dec_hi, w.rec_lo, w.rec_hi)
True
```

Other Wavelet's properties are:

Wavelet *name*, *short\_family\_name* and *family\_name*:

```
>>> print(w.name)
db3
>>> print(w.short_family_name)
db
>>> print(w.family_name)
Daubechies
```

- Decomposition (*dec\_len*) and reconstruction (*rec\_len*) filter lengths:

```
>>> int(w.dec_len) # int() is for normalizing longs and ints for doctest
6
>>> int(w.rec_len)
6
```

- Orthogonality (*orthogonal*) and biorthogonality (*biorthogonal*):

```
>>> w.orthogonal
True
>>> w.biorthogonal
True
```

- Symmetry (*symmetry*):

```
>>> print(w.symmetry)
asymmetric
```

- Number of vanishing moments for the scaling function  $\phi$  (*vanishing\_moments\_phi*) and the wavelet function  $\psi$  (*vanishing\_moments\_psi*) associated with the filters:

```
>>> w.vanishing_moments_phi
0
>>> w.vanishing_moments_psi
3
```

Now when we know a bit about the builtin Wavelets, let's see how to create *custom Wavelets* objects. These can be done in two ways:

1. Passing the filter bank object that implements the `filter_bank` attribute. The attribute must return four filters coefficients.

```
>>> class MyHaarFilterBank(object):
...     @property
...     def filter_bank(self):
...         from math import sqrt
...         return ([sqrt(2)/2, sqrt(2)/2], [-sqrt(2)/2, sqrt(2)/2],
...                 [sqrt(2)/2, sqrt(2)/2], [sqrt(2)/2, -sqrt(2)/2])

>>> my_wavelet = pywt.Wavelet('My Haar Wavelet', filter_bank=MyHaarFilterBank())
```

2. Passing the filters coefficients directly as the `filter_bank` parameter.

```
>>> from math import sqrt
>>> my_filter_bank = ([sqrt(2)/2, sqrt(2)/2], [-sqrt(2)/2, sqrt(2)/2],
...                  [sqrt(2)/2, sqrt(2)/2], [sqrt(2)/2, -sqrt(2)/2])
>>> my_wavelet = pywt.Wavelet('My Haar Wavelet', filter_bank=my_filter_bank)
```

Note that such custom wavelets **will not** have all the properties set to correct values:

```
>>> print(my_wavelet)
Wavelet My Haar Wavelet
Family name:
Short name:
Filters length: 2
Orthogonal:      False
Biorthogonal:   False
Symmetry:       unknown
DWT:            True
CWT:            False
```

You can however set a couple of them on your own:

```
>>> my_wavelet.orthogonal = True
>>> my_wavelet.biorthogonal = True
```

```
>>> print(my_wavelet)
Wavelet My Haar Wavelet
  Family name:
  Short name:
  Filters length: 2
  Orthogonal:    True
  Biorthogonal:  True
  Symmetry:     unknown
  DWT:          True
  CWT:          False
```

### And now... the `wavefun`!

We all know that the fun with wavelets is in wavelet functions. Now what would be this package without a tool to compute wavelet and scaling functions approximations?

This is the purpose of the `wavefun()` method, which is used to approximate scaling function ( $\phi$ ) and wavelet function ( $\psi$ ) at the given level of refinement, based on the filters coefficients.

The number of returned values varies depending on the wavelet's orthogonality property. For orthogonal wavelets the result is tuple with scaling function, wavelet function and xgrid coordinates.

```
>>> w = pywt.Wavelet('sym3')
>>> w.orthogonal
True
>>> (phi, psi, x) = w.wavefun(level=5)
```

For biorthogonal (non-orthogonal) wavelets different scaling and wavelet functions are used for decomposition and reconstruction, and thus five elements are returned: decomposition scaling and wavelet functions approximations, reconstruction scaling and wavelet functions approximations, and the xgrid.

```
>>> w = pywt.Wavelet('bior1.3')
>>> w.orthogonal
False
>>> (phi_d, psi_d, phi_r, psi_r, x) = w.wavefun(level=5)
```

#### See also:

You can find live examples of `wavefun()` usage and images of all the built-in wavelets on the [Wavelet Properties Browser](#) page. However, **this website is no longer actively maintained** and does not include every wavelet present in PyWavelets. The precision of the wavelet coefficients at that site is also lower than those included in PyWavelets.

## 5.3.2 Signal Extension Modes

Import `pywt` first

```
>>> import pywt
```

```
>>> def format_array(a):
...     """Consistent array representation across different systems"""
...     import numpy
...     a = numpy.where(numpy.abs(a) < 1e-5, 0, a)
...     return numpy.array2string(a, precision=5, separator=' ', suppress_small=True)
```

List of available signal extension *modes*:

```
>>> print(pywt.Modes.modes)
['zero', 'constant', 'symmetric', 'periodic', 'smooth', 'periodization', 'reflect']
```

Invalid mode name should rise a `ValueError`:

```
>>> pywt.dwt([1,2,3,4], 'db2', 'invalid')
Traceback (most recent call last):
...
ValueError: Unknown mode name 'invalid'.
```

You can also refer to modes via *Modes* class attributes:

```
>>> x = [1, 2, 1, 5, -1, 8, 4, 6]
>>> for mode_name in ['zero', 'constant', 'symmetric', 'reflect', 'periodic', 'smooth
↳', 'periodization']:
...     mode = getattr(pywt.Modes, mode_name)
...     cA, cD = pywt.dwt(x, 'db2', mode)
...     print("Mode: %d (%s)" % (mode, mode_name))
Mode: 0 (zero)
Mode: 2 (constant)
Mode: 1 (symmetric)
Mode: 6 (reflect)
Mode: 4 (periodic)
Mode: 3 (smooth)
Mode: 5 (periodization)
```

The default mode is *symmetric*:

```
>>> cA, cD = pywt.dwt(x, 'db2')
>>> print(cA)
[ 1.76776695  1.73309178  3.40612438  6.32928585  7.77817459]
>>> print(cD)
[-0.61237244 -2.15599552 -5.95034847 -1.21545369  1.22474487]
>>> print(pywt.idwt(cA, cD, 'db2'))
[ 1.  2.  1.  5. -1.  8.  4.  6.]
```

And using a keyword argument:

```
>>> cA, cD = pywt.dwt(x, 'db2', mode='symmetric')
>>> print(cA)
[ 1.76776695  1.73309178  3.40612438  6.32928585  7.77817459]
>>> print(cD)
[-0.61237244 -2.15599552 -5.95034847 -1.21545369  1.22474487]
>>> print(pywt.idwt(cA, cD, 'db2'))
[ 1.  2.  1.  5. -1.  8.  4.  6.]
```

## 5.3.3 DWT and IDWT

### Discrete Wavelet Transform

Let's do a *Discrete Wavelet Transform* of a sample data `x` using the `db2` wavelet. It's simple..

```
>>> import pywt
>>> x = [3, 7, 1, 1, -2, 5, 4, 6]
>>> cA, cD = pywt.dwt(x, 'db2')
```

And the approximation and details coefficients are in `cA` and `cD` respectively:

```
>>> print(cA)
[ 5.65685425  7.39923721  0.22414387  3.33677403  7.77817459]
>>> print(cD)
[-2.44948974 -1.60368225 -4.44140056 -0.41361256  1.22474487]
```

## Inverse Discrete Wavelet Transform

Now let's do an opposite operation - *Inverse Discrete Wavelet Transform*:

```
>>> print(pywt.idwt(cA, cD, 'db2'))
[ 3.  7.  1.  1. -2.  5.  4.  6.]
```

Voilà! That's it!

## More Examples

Now let's experiment with the `dwt()` some more. For example let's pass a *Wavelet* object instead of the wavelet name and specify signal extension mode (the default is *symmetric*) for the border effect handling:

```
>>> w = pywt.Wavelet('sym3')
>>> cA, cD = pywt.dwt(x, wavelet=w, mode='constant')
>>> print(cA)
[ 4.38354585  3.80302657  7.31813271 -0.58565539  4.09727044  7.81994027]
>>> print(cD)
[-1.33068221 -2.78795192 -3.16825651 -0.67715519 -0.09722957 -0.07045258]
```

Note that the output coefficients arrays length depends not only on the input data length but also on the `:class:Wavelet` type (particularly on its `filters` length that are used in the transformation).

To find out what will be the output data size use the `dwt_coeff_len()` function:

```
>>> # int() is for normalizing Python integers and long integers for documentation_
↳tests
>>> int(pywt.dwt_coeff_len(data_len=len(x), filter_len=w.dec_len, mode='symmetric'))
6
>>> int(pywt.dwt_coeff_len(len(x), w, 'symmetric'))
6
>>> len(cA)
6
```

Looks fine. (And if you expected that the output length would be a half of the input data length, well, that's the trade-off that allows for the perfect reconstruction...).

The third argument of the `dwt_coeff_len()` is the already mentioned signal extension mode (please refer to the PyWavelets' documentation for the *modes* description). Currently there are six *extension modes* available:

```
>>> pywt.Modes.modes
['zero', 'constant', 'symmetric', 'periodic', 'smooth', 'periodization', 'reflect']
```

As you see in the above example, the *periodization* (periodization) mode is slightly different from the others. It's aim when doing the *DWT* transform is to output coefficients arrays that are half of the length of the input data.

Knowing that, you should never mix the periodization mode with other modes when doing *DWT* and *IDWT*. Otherwise, it will produce **invalid results**:

```
>>> x
[3, 7, 1, 1, -2, 5, 4, 6]
>>> cA, cD = pywt.dwt(x, wavelet=w, mode='periodization')
>>> print(pywt.idwt(cA, cD, 'sym3', 'symmetric')) # invalid mode
[ 1.  1. -2.  5.]
>>> print(pywt.idwt(cA, cD, 'sym3', 'periodization'))
[ 3.  7.  1.  1. -2.  5.  4.  6.]
```

## Tips & tricks

### Passing None instead of coefficients data to idwt ()

Now some tips & tricks. Passing None as one of the coefficient arrays parameters is similar to passing a *zero-filled* array. The results are simply the same:

```
>>> print(pywt.idwt([1,2,0,1], None, 'db2', 'symmetric'))
[ 1.19006969  1.54362308  0.44828774 -0.25881905  0.48296291  0.8365163 ]
```

```
>>> print(pywt.idwt([1, 2, 0, 1], [0, 0, 0, 0], 'db2', 'symmetric'))
[ 1.19006969  1.54362308  0.44828774 -0.25881905  0.48296291  0.8365163 ]
```

```
>>> print(pywt.idwt(None, [1, 2, 0, 1], 'db2', 'symmetric'))
[ 0.57769726 -0.93125065  1.67303261 -0.96592583 -0.12940952 -0.22414387]
```

```
>>> print(pywt.idwt([0, 0, 0, 0], [1, 2, 0, 1], 'db2', 'symmetric'))
[ 0.57769726 -0.93125065  1.67303261 -0.96592583 -0.12940952 -0.22414387]
```

Remember that only one argument at a time can be None:

```
>>> print(pywt.idwt(None, None, 'db2', 'symmetric'))
Traceback (most recent call last):
...
ValueError: At least one coefficient parameter must be specified.
```

### Coefficients data size in idwt

When doing the *IDWT* transform, usually the coefficient arrays must have the same size.

```
>>> print(pywt.idwt([1, 2, 3, 4, 5], [1, 2, 3, 4], 'db2', 'symmetric'))
Traceback (most recent call last):
...
ValueError: Coefficients arrays must have the same size.
```

Not every coefficient array can be used in *IDWT*. In the following example the *idwt ()* will fail because the input arrays are invalid - they couldn't be created as a result of *DWT*, because the minimal output length for *dwt* using *db4* wavelet and the *symmetric* mode is 4, not 3:

```
>>> pywt.idwt([1,2,4], [4,1,3], 'db4', 'symmetric')
Traceback (most recent call last):
...
ValueError: Invalid coefficient arrays length for specified wavelet. Wavelet and mode_
↳ must be the same as used for decomposition.
```

```
>>> int(pywt.dwt_coeff_len(1, pywt.Wavelet('db4').dec_len, 'symmetric'))
4
```

### 5.3.4 Multilevel DWT, IDWT and SWT

#### Multilevel DWT decomposition

```
>>> import pywt
>>> x = [3, 7, 1, 1, -2, 5, 4, 6]
>>> db1 = pywt.Wavelet('db1')
>>> cA3, cD3, cD2, cD1 = pywt.wavedec(x, db1)
>>> print(cA3)
[ 8.83883476]
>>> print(cD3)
[-0.35355339]
>>> print(cD2)
[ 4.  -3.5]
>>> print(cD1)
[-2.82842712  0.          -4.94974747 -1.41421356]
```

```
>>> pywt.dwt_max_level(len(x), db1)
3
```

```
>>> cA2, cD2, cD1 = pywt.wavedec(x, db1, mode='constant', level=2)
```

#### Multilevel IDWT reconstruction

```
>>> coeffs = pywt.wavedec(x, db1)
>>> print(pywt.waverec(coeffs, db1))
[ 3.  7.  1.  1. -2.  5.  4.  6.]
```

#### Multilevel SWT decomposition

```
>>> x = [3, 7, 1, 3, -2, 6, 4, 6]
>>> (cA2, cD2), (cA1, cD1) = pywt.swt(x, db1, level=2)
>>> print(cA1)
[ 7.07106781  5.65685425  2.82842712  0.70710678  2.82842712  7.07106781
  7.07106781  6.36396103]
>>> print(cD1)
[-2.82842712  4.24264069 -1.41421356  3.53553391 -5.65685425  1.41421356
 -1.41421356  2.12132034]
>>> print(cA2)
[ 7.   4.5  4.   5.5  7.   9.5 10.   8.5]
>>> print(cD2)
[ 3.   3.5  0.  -4.5 -3.   0.5  0.   0.5]
```

```
>>> [(cA2, cD2)] = pywt.swt(cA1, db1, level=1, start_level=1)
>>> print(cA2)
[ 7.   4.5  4.   5.5  7.   9.5 10.   8.5]
>>> print(cD2)
[ 3.   3.5  0.  -4.5 -3.   0.5  0.   0.5]
```

```
>>> coeffs = pywt.swt(x, db1)
>>> len(coeffs)
3
>>> pywt.swt_max_level(len(x))
3
```

```
>>> from __future__ import print_function
```

## 5.3.5 Wavelet Packets

### Import pywt

```
>>> import pywt
```

```
>>> def format_array(a):
...     """Consistent array representation across different systems"""
...     import numpy
...     a = numpy.where(numpy.abs(a) < 1e-5, 0, a)
...     return numpy.array2string(a, precision=5, separator=' ', suppress_small=True)
```

### Create Wavelet Packet structure

Ok, let's create a sample *WaveletPacket*:

```
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

The input data and decomposition coefficients are stored in the `WaveletPacket.data` attribute:

```
>>> print(wp.data)
[1, 2, 3, 4, 5, 6, 7, 8]
```

*Nodes* are identified by paths. For the root node the path is `' '` and the decomposition level is 0.

```
>>> print(repr(wp.path))
''
>>> print(wp.level)
0
```

The `maxlevel`, if not given as param in the constructor, is automatically computed:

```
>>> print(wp['ad'].maxlevel)
3
```

### Traversing WP tree:

#### Accessing subnodes:

```
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

First check what is the maximum level of decomposition:

```
>>> print(wp.maxlevel)
3
```

and try accessing subnodes of the WP tree:

- 1st level:

```
>>> print(wp['a'].data)
[ 2.12132034  4.94974747  7.77817459 10.60660172]
>>> print(wp['a'].path)
a
```

- 2nd level:

```
>>> print(wp['aa'].data)
[ 5. 13.]
>>> print(wp['aa'].path)
aa
```

- 3rd level:

```
>>> print(wp['aaa'].data)
[ 12.72792206]
>>> print(wp['aaa'].path)
aaa
```

Ups, we have reached the maximum level of decomposition and got an `IndexError`:

```
>>> print(wp['aaaa'].data)
Traceback (most recent call last):
...
IndexError: Path length is out of range.
```

Now try some invalid path:

```
>>> print(wp['ac'])
Traceback (most recent call last):
...
ValueError: Subnode name must be in ['a', 'd'], not 'c'.
```

which just yielded a `ValueError`.

### Accessing Node's attributes:

`WaveletPacket` object is a tree data structure, which evaluates to a set of `Node` objects. `WaveletPacket` is just a special subclass of the `Node` class (which in turn inherits from the `BaseNode`).

Tree nodes can be accessed using the `obj[x]` (`Node.__getitem__()`) operator. Each tree node has a set of attributes: `data`, `path`, `node_name`, `parent`, `level`, `maxlevel` and `mode`.

```
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

```
>>> print(wp['ad'].data)
[-2. -2.]
```

```
>>> print(wp['ad'].path)
ad
```

```
>>> print(wp['ad'].node_name)
d
```

```
>>> print(wp['ad'].parent.path)
a
```

```
>>> print(wp['ad'].level)
2
```

```
>>> print(wp['ad'].maxlevel)
3
```

```
>>> print(wp['ad'].mode)
symmetric
```

### Collecting nodes

```
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

We can get all nodes on the particular level either in natural order:

```
>>> print([node.path for node in wp.get_level(3, 'natural')])
['aaa', 'aad', 'ada', 'add', 'daa', 'dad', 'dda', 'ddd']
```

or sorted based on the band frequency (freq):

```
>>> print([node.path for node in wp.get_level(3, 'freq')])
['aaa', 'aad', 'add', 'ada', 'dda', 'ddd', 'dad', 'daa']
```

Note that `WaveletPacket.get_level()` also performs automatic decomposition until it reaches the specified level.

### Reconstructing data from Wavelet Packets:

```
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

Now create a new *Wavelet Packet* and set its nodes with some data.

```
>>> new_wp = pywt.WaveletPacket(data=None, wavelet='db1', mode='symmetric')
```

```
>>> new_wp['aa'] = wp['aa'].data
>>> new_wp['ad'] = [-2., -2.]
```

For convenience, `Node.data` gets automatically extracted from the *Node* object:

```
>>> new_wp['d'] = wp['d']
```

And reconstruct the data from the aa, ad and d packets.

```
>>> print(new_wp.reconstruct(update=False))
[ 1.  2.  3.  4.  5.  6.  7.  8.]
```

If the update param in the reconstruct method is set to False, the node's data will not be updated.

```
>>> print(new_wp.data)
None
```

Otherwise, the data attribute will be set to the reconstructed value.

```
>>> print(new_wp.reconstruct(update=True))
[ 1.  2.  3.  4.  5.  6.  7.  8.]
>>> print(new_wp.data)
[ 1.  2.  3.  4.  5.  6.  7.  8.]
```

```
>>> print([n.path for n in new_wp.get_leaf_nodes(False)])
['aa', 'ad', 'd']
```

```
>>> print([n.path for n in new_wp.get_leaf_nodes(True)])
['aaa', 'aad', 'ada', 'add', 'daa', 'dad', 'dda', 'ddd']
```

### Removing nodes from Wavelet Packet tree:

Let's create a sample data:

```
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

First, start with a tree decomposition at level 2. Leaf nodes in the tree are:

```
>>> dummy = wp.get_level(2)
>>> for n in wp.get_leaf_nodes(False):
...     print(n.path, format_array(n.data))
aa [ 5. 13.]
ad [-2. -2.]
da [-1. -1.]
dd [ 0.  0.]
```

```
>>> node = wp['ad']
>>> print(node)
ad: [-2. -2.]
```

To remove a node from the WP tree, use Python's `del obj[x]` (`Node.__delitem__`):

```
>>> del wp['ad']
```

The leaf nodes that left in the tree are:

```
>>> for n in wp.get_leaf_nodes():
...     print(n.path, format_array(n.data))
```

(continues on next page)

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```
aa [ 5. 13.]
da [-1. -1.]
dd [ 0.  0.]
```

And the reconstruction is:

```
>>> print(wp.reconstruct())
[ 2.  3.  2.  3.  6.  7.  6.  7.]
```

Now restore the deleted node value.

```
>>> wp['ad'].data = node.data
```

Printing leaf nodes and tree reconstruction confirms the original state of the tree:

```
>>> for n in wp.get_leaf_nodes(False):
...     print(n.path, format_array(n.data))
aa [ 5. 13.]
ad [-2. -2.]
da [-1. -1.]
dd [ 0.  0.]
```

```
>>> print(wp.reconstruct())
[ 1.  2.  3.  4.  5.  6.  7.  8.]
```

### Lazy evaluation:

---

**Note:** This section is for demonstration of pywt internals purposes only. Do not rely on the attribute access to nodes as presented in this example.

---

```
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

1. At first the wp's attribute a is None

```
>>> print(wp.a)
None
```

**Remember that you should not rely on the attribute access.**

2. At first attempt to access the node it is computed via decomposition of its parent node (the wp object itself).

```
>>> print(wp['a'])
a: [ 2.12132034  4.94974747  7.77817459 10.60660172]
```

3. Now the wp.a is set to the newly created node:

```
>>> print(wp.a)
a: [ 2.12132034  4.94974747  7.77817459 10.60660172]
```

And so is wp.d:

```
>>> print(wp.d)
d: [-0.70710678 -0.70710678 -0.70710678 -0.70710678]
```

### 5.3.6 2D Wavelet Packets

#### Import pywt

```
>>> from __future__ import print_function
>>> import pywt
>>> import numpy
```

#### Create 2D Wavelet Packet structure

Start with preparing test data:

```
>>> x = numpy.array([[1, 2, 3, 4, 5, 6, 7, 8]] * 8, 'd')
>>> print(x)
[[ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
```

Now create a *2D Wavelet Packet* object:

```
>>> wp = pywt.WaveletPacket2D(data=x, wavelet='db1', mode='symmetric')
```

The input data and decomposition coefficients are stored in the *WaveletPacket2D.data* attribute:

```
>>> print(wp.data)
[[ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
```

*Nodes* are identified by paths. For the root node the path is '' and the decomposition level is 0.

```
>>> print(repr(wp.path))
''
>>> print(wp.level)
0
```

The *WaveletPacket2D.maxlevel*, if not given in the constructor, is automatically computed based on the data size:

```
>>> print(wp.maxlevel)
3
```

## Traversing WP tree:

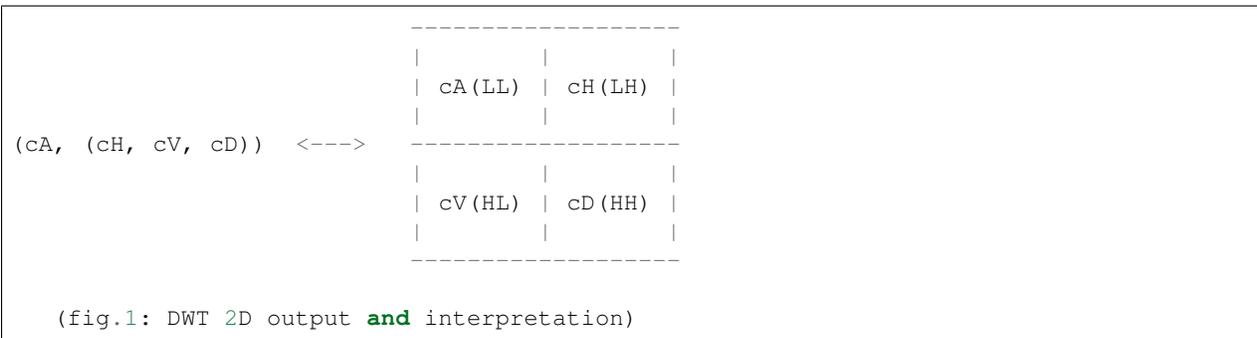
Wavelet Packet *nodes* are arranged in a tree. Each node in a WP tree is uniquely identified and addressed by a path string.

In the 1D *WaveletPacket* case nodes were accessed using 'a' (approximation) and 'd' (details) path names (each node has two 1D children).

Because now we deal with a bit more complex structure (each node has four children), we have four basic path names based on the dwt 2D output convention to address the WP2D structure:

- a - LL, low-low coefficients
- h - LH, low-high coefficients
- v - HL, high-low coefficients
- d - HH, high-high coefficients

In other words, subnode naming corresponds to the *dwt2()* function output naming convention (as wavelet packet transform is based on the dwt2 transform):



Knowing what the nodes names are, we can now access them using the indexing operator `obj[x]` (*WaveletPacket2D.\_\_getitem\_\_()*):

```

>>> print(wp['a'].data)
[[ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]]
>>> print(wp['h'].data)
[[ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]]
>>> print(wp['v'].data)
[[-1. -1. -1. -1.]
 [-1. -1. -1. -1.]
 [-1. -1. -1. -1.]
 [-1. -1. -1. -1.]]
>>> print(wp['d'].data)
[[ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]]
  
```

Similarly, a subnode of a subnode can be accessed by:

```
>>> print(wp['aa'].data)
[[ 10.  26.]
 [ 10.  26.]]
```

Indexing base *WaveletPacket2D* (as well as 1D *WaveletPacket*) using compound path is just the same as indexing WP subnode:

```
>>> node = wp['a']
>>> print(node['a'].data)
[[ 10.  26.]
 [ 10.  26.]]
>>> print(wp['a']['a'].data is wp['aa'].data)
True
```

Following down the decomposition path:

```
>>> print(wp['aaa'].data)
[[ 36.]]
>>> print(wp['aaaa'].data)
Traceback (most recent call last):
...
IndexError: Path length is out of range.
```

Ups, we have reached the maximum level of decomposition for the 'aaaa' path, which btw. was:

```
>>> print(wp.maxlevel)
3
```

Now try some invalid path:

```
>>> print(wp['f'])
Traceback (most recent call last):
...
ValueError: Subnode name must be in ['a', 'h', 'v', 'd'], not 'f'.
```

### Accessing Node2D's attributes:

*WaveletPacket2D* is a tree data structure, which evaluates to a set of *Node2D* objects. *WaveletPacket2D* is just a special subclass of the *Node2D* class (which in turn inherits from a *BaseNode*, just like with *Node* and *WaveletPacket* for the 1D case.).

```
>>> print(wp['av'].data)
[[-4. -4.]
 [-4. -4.]]
```

```
>>> print(wp['av'].path)
av
```

```
>>> print(wp['av'].node_name)
v
```

```
>>> print(wp['av'].parent.path)
a
```

```
>>> print(wp['av'].parent.data)
[[ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]]
```

```
>>> print(wp['av'].level)
2
```

```
>>> print(wp['av'].maxlevel)
3
```

```
>>> print(wp['av'].mode)
symmetric
```

## Collecting nodes

We can get all nodes on the particular level using the `WaveletPacket2D.get_level()` method:

- 0 level - the root `wp` node:

```
>>> len(wp.get_level(0))
1
>>> print([node.path for node in wp.get_level(0)])
['']
```

- 1st level of decomposition:

```
>>> len(wp.get_level(1))
4
>>> print([node.path for node in wp.get_level(1)])
['a', 'h', 'v', 'd']
```

- 2nd level of decomposition:

```
>>> len(wp.get_level(2))
16
>>> paths = [node.path for node in wp.get_level(2)]
>>> for i, path in enumerate(paths):
...     if (i+1) % 4 == 0:
...         print(path)
...     else:
...         print(path, end=' ')
aa ah av ad
ha hh hv hd
va vh vv vd
da dh dv dd
```

- 3rd level of decomposition:

```
>>> print(len(wp.get_level(3)))
64
>>> paths = [node.path for node in wp.get_level(3)]
>>> for i, path in enumerate(paths):
```

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```

...     if (i+1) % 8 == 0:
...         print(path)
...     else:
...         print(path, end=' ')
aaa aah aav aad aha ahh ahv ahd
ava avh avv avd ada adh adv add
haa hah hav had hha hhh hhv hhd
hva hvh hvv hvd hda hdh hdv hdd
vaa vah vav vad vha vhh vhv vhd
vva vvh vvv vvd vda vdh vdv vdd
daa dah dav dad dha dhh dhv dhd
dva dvh dvv dvd dda ddh ddv ddd

```

Note that `WaveletPacket2D.get_level()` performs automatic decomposition until it reaches the given level.

### Reconstructing data from Wavelet Packets:

Let's create a new empty 2D Wavelet Packet structure and set its nodes values with known data from the previous examples:

```
>>> new_wp = pywt.WaveletPacket2D(data=None, wavelet='db1', mode='symmetric')
```

```
>>> new_wp['vh'] = wp['vh'].data # [[0.0, 0.0], [0.0, 0.0]]
>>> new_wp['vv'] = wp['vh'].data # [[0.0, 0.0], [0.0, 0.0]]
>>> new_wp['vd'] = [[0.0, 0.0], [0.0, 0.0]]

```

```
>>> new_wp['a'] = [[3.0, 7.0, 11.0, 15.0], [3.0, 7.0, 11.0, 15.0],
...               [3.0, 7.0, 11.0, 15.0], [3.0, 7.0, 11.0, 15.0]]
>>> new_wp['d'] = [[0.0, 0.0, 0.0, 0.0], [0.0, 0.0, 0.0, 0.0],
...               [0.0, 0.0, 0.0, 0.0], [0.0, 0.0, 0.0, 0.0]]

```

For convenience, `Node2D.data` gets automatically extracted from the base `Node2D` object:

```
>>> new_wp['h'] = wp['h'] # all zeros

```

Note: just remember to not assign to the `node.data` parameter directly (todo).

And reconstruct the data from the `a`, `d`, `vh`, `vv`, `vd` and `h` packets (Note that `va` node was not set and the WP tree is “not complete” - the `va` branch will be treated as *zero-array*):

```
>>> print(new_wp.reconstruct(update=False))
[[ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]]

```

Now set the `va` node with the known values and do the reconstruction again:

```
>>> new_wp['va'] = wp['va'].data # [[-2.0, -2.0], [-2.0, -2.0]]
>>> print(new_wp.reconstruct(update=False))
[[ 1.  2.  3.  4.  5.  6.  7.  8.]

```

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```
[ 1.  2.  3.  4.  5.  6.  7.  8.]
[ 1.  2.  3.  4.  5.  6.  7.  8.]
[ 1.  2.  3.  4.  5.  6.  7.  8.]
[ 1.  2.  3.  4.  5.  6.  7.  8.]
[ 1.  2.  3.  4.  5.  6.  7.  8.]
[ 1.  2.  3.  4.  5.  6.  7.  8.]
[ 1.  2.  3.  4.  5.  6.  7.  8.]
```

which is just the same as the base sample data  $x$ .

Of course we can go the other way and remove nodes from the tree. If we delete the `va` node, again, we get the “not complete” tree from one of the previous examples:

```
>>> del new_wp['va']
>>> print(new_wp.reconstruct(update=False))
[[ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]]
```

Just restore the node before next examples.

```
>>> new_wp['va'] = wp['va'].data
```

If the `update` param in the `WaveletPacket2D.reconstruct()` method is set to `False`, the node’s `Node2D.data` attribute will not be updated.

```
>>> print(new_wp.data)
None
```

Otherwise, the `WaveletPacket2D.data` attribute will be set to the reconstructed value.

```
>>> print(new_wp.reconstruct(update=True))
[[ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]]
>>> print(new_wp.data)
[[ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]]
```

Since we have an interesting WP structure built, it is a good occasion to present the `WaveletPacket2D.get_leaf_nodes()` method, which collects non-zero leaf nodes from the WP tree:

```
>>> print([n.path for n in new_wp.get_leaf_nodes()])
['a', 'h', 'va', 'vh', 'vv', 'vd', 'd']
```

Passing the `decompose = True` parameter to the method will force the WP object to do a full decomposition up to the *maximum level* of decomposition:

```
>>> paths = [n.path for n in new_wp.get_leaf_nodes(decompose=True)]
>>> len(paths)
64
>>> for i, path in enumerate(paths):
...     if (i+1) % 8 == 0:
...         print(path)
...     else:
...         try:
...             print(path, end=' ')
...         except:
...             print(path, end=' ')
aaa aah aav aad aha ahh ahv ahd
ava avh avv avd ada adh adv add
haa hah hav had hha hhh hhv hhd
hva hvh hvv hvd hda hdh hdv hdd
vaa vah vav vad vha vhh vhw vhd
vva vvh vvv vvd vda vdh vdv vdd
daa dah dav dad dha dhh dhv dhd
dva dvh dvv dvd dda ddh ddv ddd
```

### Lazy evaluation:

**Note:** This section is for demonstration of pywt internals purposes only. Do not rely on the attribute access to nodes as presented in this example.

```
>>> x = numpy.array([[1, 2, 3, 4, 5, 6, 7, 8]] * 8)
>>> wp = pywt.WaveletPacket2D(data=x, wavelet='db1', mode='symmetric')
```

1. At first the wp's attribute `a` is `None`

```
>>> print(wp.a)
None
```

**Remember that you should not rely on the attribute access.**

2. During the first attempt to access the node it is computed via decomposition of its parent node (the wp object itself).

```
>>> print(wp['a'])
a: [[ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]]
```

3. Now the `a` is set to the newly created node:

```
>>> print(wp.a)
a: [[ 3.  7. 11. 15.]
```

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```
[ 3.  7. 11. 15.]
[ 3.  7. 11. 15.]
[ 3.  7. 11. 15.]
```

And so is `wp.d`:

```
>>> print(wp.d)
d: [[ 0.  0.  0.  0.]
     [ 0.  0.  0.  0.]
     [ 0.  0.  0.  0.]
     [ 0.  0.  0.  0.]
```

### 5.3.7 Gotchas

PyWavelets utilizes NumPy under the hood. That's why handling the data containing None values can be surprising. None values are converted to 'not a number' (`numpy.NaN`) values:

```
>>> import numpy, pywt
>>> x = [None, None]
>>> mode = 'symmetric'
>>> wavelet = 'db1'
>>> cA, cD = pywt.dwt(x, wavelet, mode)
>>> numpy.all(numpy.isnan(cA))
True
>>> numpy.all(numpy.isnan(cD))
True
>>> rec = pywt.idwt(cA, cD, wavelet, mode)
>>> numpy.all(numpy.isnan(rec))
True
```

## 5.4 Contributing

All contributions including bug reports, bug fixes, new feature implementations and documentation improvements are welcome. Moreover, developers with an interest in PyWavelets are very welcome to join the development team! Please see our [guidelines for pull requests](#) for more information.

Contributors are expected to behave in a productive and respectful manner in accordance with our [community guidelines](#).

### 5.4.1 History

PyWavelets started in 2006 as an academic project for a masters thesis on *Analysis and Classification of Medical Signals using Wavelet Transforms* and was maintained until 2012 by its [original developer](#). In 2013 maintenance was taken over in a [new repo](#)) by a larger development team - a move supported by the original developer. The repo move doesn't mean that this is a fork - the package continues to be developed under the name "PyWavelets", and released on PyPI and Github (see [this issue](#) for the discussion where that was decided).

## 5.5 Development guide

This section contains information on building and installing PyWavelets from source code as well as instructions for preparing the build environment on Windows and Linux.

### 5.5.1 Preparing Windows build environment

To start developing PyWavelets code on Windows you will have to install a C compiler and prepare the build environment.

#### Installing Windows SDK C/C++ compiler

Depending on your Python version, a different version of the Microsoft Visual C++ compiler will be required to build extensions. The same compiler that was used to build Python itself should be used.

For official binary builds of Python 2.6 to 3.2, this will be VS 2008. Python 3.3 and 3.4 were compiled with VS 2010, and for Python 3.5 it will be MSVC 2015.

The MSVC version should be printed when starting a Python REPL, and can be checked against the note below:

**Note:** For reference:

- the *MSC v.1500* in the Python version string is Microsoft Visual C++ 2008 (Microsoft Visual Studio 9.0 with msvc90.dll runtime)
- *MSC v.1600* is MSVC 2010 (10.0 with msvc100.dll runtime)
- *MSC v.1700* is MSVC 2012 (11.0)
- *MSC v.1800* is MSVC 2013 (12.0)
- *MSC v.1900* is MSVC 2015 (14.0)

```
Python 2.7.3 (default, Apr 10 2012, 23:31:26) [MSC v.1500 32 bit (Intel)] on win32
Python 3.2 (r32:88445, Feb 20 2011, 21:30:00) [MSC v.1500 64 bit (AMD64)] on win32
```

To get started first download, extract and install *Microsoft Windows SDK for Windows 7 and .NET Framework 3.5 SP1* from <http://www.microsoft.com/downloads/en/details.aspx?familyid=71DEB800-C591-4F97-A900-BEA146E4FAE1&displaylang=en>.

There are several ISO images on the site, so just grab the one that is suitable for your platform:

- GRMSDK\_EN\_DVD.iso for 32-bit x86 platform
- GRMSDKX\_EN\_DVD.iso for 64-bit AMD64 platform (AMD64 is the codename for 64-bit CPU architecture, not the processor manufacturer)

After installing the SDK and before compiling the extension you have to configure some environment variables.

For 32-bit build execute the util/setenv\_build32.bat script in the cmd window:

```
rem Configure the environment for 32-bit builds.
rem Use "vcvars32.bat" for a 32-bit build.
"C:\Program Files (x86)\Microsoft Visual Studio 9.0\VC\bin\vcvars32.bat"
rem Convince setup.py to use the SDK tools.
set MSSdk=1
```

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```
setenv /x86 /release
set DISTUTILS_USE_SDK=1
```

For 64-bit use `util/setenv_build64.bat`:

```
rem Configure the environment for 64-bit builds.
rem Use "vcvars32.bat" for a 32-bit build.
"C:\Program Files (x86)\Microsoft Visual Studio 9.0\VC\bin\vcvars64.bat"
rem Convince setup.py to use the SDK tools.
set MSSdk=1
setenv /x64 /release
set DISTUTILS_USE_SDK=1
```

See also <http://wiki.cython.org/64BitCythonExtensionsOnWindows>.

## MinGW C/C++ compiler

MinGW distribution can be downloaded from <http://sourceforge.net/projects/mingwbuilds/>.

In order to change the settings and use MinGW as the default compiler, edit or create a Distutils configuration file `c:\Python2*\Lib\distutils\distutils.cfg` and place the following entry in it:

```
[build]
compiler = mingw32
```

You can also take a look at Cython's "Installing MinGW on Windows" page at <http://wiki.cython.org/InstallingOnWindows> for more info.

---

**Note:** Python 2.7/3.2 distutils package is incompatible with the current version (4.7+) of MinGW (MinGW dropped the `-mno-cygwin` flag, which is still passed by distutils).

To use MinGW to compile Python extensions you have to patch the `distutils/cygwinccompiler.py` library module and remove every occurrence of `-mno-cygwin`.

See <http://bugs.python.org/issue12641> bug report for more information on the issue.

---

## Next steps

After completing these steps continue with *Installing build dependencies*.

## 5.5.2 Preparing Linux build environment

There is a good chance that you already have a working build environment. Just skip steps that you don't need to execute.

### Installing basic build tools

Note that the example below uses `aptitude` package manager, which is specific to Debian and Ubuntu Linux distributions. Use your favourite package manager to install these packages on your OS.

```
aptitude install build-essential gcc python-dev git-core
```

## Next steps

After completing these steps continue with *Installing build dependencies*.

## 5.5.3 Installing build dependencies

### Setting up Python virtual environment

A good practice is to create a separate Python virtual environment for each project. If you don't have `virtualenv` yet, install and activate it using:

```
curl -O https://raw.githubusercontent.com/pypa/virtualenv/master/virtualenv.py
python virtualenv.py <name_of_the_venv>
. <name_of_the_venv>/bin/activate
```

### Installing Cython

Use `pip` (<http://pypi.python.org/pypi/pip>) to install Cython:

```
pip install Cython>=0.16
```

### Installing numpy

Use `pip` to install `numpy`:

```
pip install numpy
```

Numpy can also be obtained via scientific python distributions such as:

- Anaconda
- Enthought Canopy
- Python(x,y)

---

**Note:** You can find binaries for 64-bit Windows on <http://www.lfd.uci.edu/~gohlke/pythonlibs/>.

---

### Installing Sphinx

`Sphinx` is a documentation tool that converts reStructuredText files into nicely looking html documentation. Install it with:

```
pip install Sphinx
```

`numpydoc` is used to format the API documentation appropriately. Install it via:

```
pip install numpydoc
```

## 5.5.4 Building and installing PyWavelets

### Installing from source code

Go to <https://github.com/PyWavelets/pywt> GitHub project page, fork and clone the repository or use the upstream repository to get the source code:

```
git clone https://github.com/PyWavelets/pywt.git PyWavelets
```

Activate your Python virtual environment, go to the cloned source directory and type the following commands to build and install the package:

```
python setup.py build
python setup.py install
```

To verify the installation run the following command:

```
python setup.py test
```

To build docs:

```
cd doc
make html
```

### Installing a development version

You can also install directly from the source repository:

```
pip install -e git+https://github.com/PyWavelets/pywt.git#egg=PyWavelets
```

or:

```
pip install PyWavelets==dev
```

### Installing a regular release from PyPi

A regular release can be installed with pip or easy\_install:

```
pip install PyWavelets
```

## 5.5.5 Testing

### Continous integration with Travis-CI

The project is using [Travis-CI](#) service for continuous integration and testing.

Current build status is:  If you are submitting a patch or pull request please make sure it does not break the build.

## Running tests locally

Tests are implemented with `nose`, so use one of:

```
$ nosetests pywt
```

```
>>> pywt.test()
```

Note doctests require `Matplotlib` in addition to the usual dependencies.

## Running tests with Tox

There's also a config file for running tests with `Tox` (`pip install tox`). To for example run tests for Python 2.7 and Python 3.4 use:

```
tox -e py27,py34
```

For more information see the `Tox` documentation.

## 5.5.6 Guidelines for Releasing PyWavelets

The following are guidelines for preparing a release of PyWavelets. The notation `vX.X.X` in the commands below would be replaced by the actual release number.

### Updating the release notes

Prior to the release, make sure the release notes are up to date. The author lists can be generated via:

```
python ./util/authors.py vP.P.P..
```

where `vP.P.P` is the previous release number.

The lists of issues closed and PRs merged can be generated via (script requires Python 2.X to run):

```
python ./util/gh_lists.py vX.X.X
```

### Tag the release

Change `ISRELEASED` to `True` in `setup.py` and commit.

Tag the release via:

```
git tag -s vX.X.X
```

Then push the `vX.X.X` tag to the PyWavelets GitHub repo.

Note that while Appveyor will build wheels for Windows, it is preferred to get those wheels from the step below. Instructions for grabbing Appveyor wheels manually here for reference only: if the commit with `ISRELEASED=True` is submitted as a PR, the wheels can be downloaded from Appveyor once it has run on the PR. They can be found under the “Artifacts” tab in the Appveyor interface.

### Build Windows, OS X and Linux wheels and upload to PyPI

Push a commit with the new tag and updates of dependency versions where needed to <https://github.com/MacPython/pywavelets-wheels>. The wheels will be produced automatically and uploaded to <http://wheels.scipy.org/>. From there they can be uploaded to PyPI automatically with `wheel-uploader`.

See the README on <https://github.com/MacPython/pywavelets-wheels> for more details.

### Create the source distribution

Remove untracked files and directories with `git clean`. *Warning: this will delete files & directories that are not under version control so you may want to do a dry run first by adding `-n`, so you can see what will be removed:*

```
git clean -x fdn
```

Then run without `-n`:

```
git clean -x fd
```

Create the source distribution files via:

```
python setup.py sdist --formats=gztar,zip
```

### Upload the release to PyPI

The binary Windows wheels downloaded from Appveyor (see above) should also be placed into the `/dist` subfolder along with the sdist archives.

The wheels and source distributions created above can all be securely uploaded to `pypi.python.org` using `twine`:

```
twine upload -s dist/*
```

Note that the documentation on ReadTheDocs (<http://pywavelets.readthedocs.org>) will have been automatically generated, so no actions need to be taken for documentation.

### Update conda-forge

Send a PR with the new version number and sha256 hash of the source release to <https://github.com/conda-forge/pywavelets-feedstock>.

### Create the release on GitHub

On the project's GitHub page, click the releases tab and then press the “*Draft a new release*” button to create a release from the appropriate tag.

### Announcing the release

Send release announcements to:

- [pywavelets@googlegroups.com](mailto:pywavelets@googlegroups.com)
- [python-announce-list@python.org](mailto:python-announce-list@python.org)
- [scipy-user@python.org](mailto:scipy-user@python.org)

## Prepare for continued development

Increment the version number in `setup.py` and change `ISRELEASED` to `False`.

Prepare new release note files for the upcoming release:

```
git add doc/release/X.X.X-notes.rst
git add doc/source/release.X.X.X.rst
```

And add `release.X.X.X` to the list in `doc/source/releasenotes.rst`

## 5.5.7 Something not working?

If these instructions are not clear or you need help setting up your development environment, go ahead and ask on the PyWavelets discussion group at <http://groups.google.com/group/pywavelets> or open a ticket on [GitHub](#).

## 5.6 Release Notes

### 5.6.1 PyWavelets 0.3.0 Release Notes

#### Contents

- *PyWavelets 0.3.0 Release Notes*
  - *New features*
    - \* *Test suite*
    - \* *n-D Inverse Discrete Wavelet Transform*
    - \* *Thresholding*
  - *Backwards incompatible changes*
  - *Other changes*
  - *Authors*
    - \* *Issues closed for v0.3.0*
    - \* *Pull requests for v0.3.0*

PyWavelets 0.3.0 is the first release of the package in 3 years. It is the result of a significant effort of a growing development team to modernize the package, to provide Python 3.x support and to make a start with providing new features as well as improved performance. A 0.4.0 release will follow shortly, and will contain more significant new features as well as changes/deprecations to streamline the API.

This release requires Python 2.6, 2.7 or 3.3-3.5 and NumPy 1.6.2 or greater.

Highlights of this release include:

- Support for Python 3.x ( $\geq 3.3$ )
- Added a test suite (based on nose, coverage up to 61% so far)
- Maintenance work: C style complying to the Numpy style guide, improved templating system, more complete docstrings, pep8/pyflakes compliance, and more.

### New features

#### Test suite

The test suite can be run with `nosetests pywt` or with:

```
>>> import pywt
>>> pywt.test()
```

#### n-D Inverse Discrete Wavelet Transform

The function `pywt.idwt_n`, which provides n-dimensional inverse DWT, has been added. It complements `idwt`, `idwt2` and `dwt_n`.

#### Thresholding

The function `pywt.threshold` has been added. It unifies the four thresholding functions that are still provided in the `pywt.thresholding` namespace.

#### Backwards incompatible changes

None in this release.

#### Other changes

Development has moved to [a new repo](#). Everyone with an interest in wavelets is welcome to contribute!

Building wheels, building with `python setup.py develop` and many other standard ways to build and install PyWavelets are supported now.

#### Authors

- Ankit Agrawal +
- François Boulogne +
- Ralf Gommers +
- David Menéndez Hurtado +
- Gregory R. Lee +
- David McInnis +
- Helder Oliveira +
- Filip Wasilewski
- Kai Wohlfahrt +

A total of 9 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

### Issues closed for v0.3.0

- #3: Remove numerix compat layer
- #4: Add single code base Python 3 support
- #5: PEP8 issues
- #6: Migrate tests to nose
- #7: Expand test coverage without Matlab to a reasonable level
- #8: Replace custom C templates by Numpy's templating system
- #9: Replace Cython templates by fused types
- #10: Replace use of `__array_interface__` with Cython's memoryviews
- #11: Format existing docstrings in numpydoc format.
- #12: Complete docstrings, they're quite sparse right now
- #13: Reorganize source tree
- #24: doc/source/regression should be moved
- #27: Broken test: test\_swt\_decomposition
- #28: Install issue, no module tools.six
- #29: wp.update fails after removal of nodes
- #32: wp.update fails on 2D
- #34: Wavelet string attributes shouldn't be bytes in Python 3
- #35: Re-enable float32 support
- #36: wavelet instance vs string
- #40: Test with Numpy 1.8rc1
- #45: demos should be updated and integrated in docs
- #60: Moving pywt forward faster
- #61: issues to address in moving towards 0.3.0
- #71: BUG: `_pywt.downcoef` always returns level=1 result

### Pull requests for v0.3.0

- #1: travis: check all branches + fix URL
- #17: [DOC] docstrings for multilevel functions
- #18: DOC: format -> functions.py
- #20: MAINT: remove unnecessary `zero()` `copy()`
- #21: Doc wavelet\_packets
- #22: Minor doc fixes
- #25: TEST: remove useless functions and use numpy instead
- #26: Merge most recent work

- #30: Adding test for wp.rst
- #41: Change to Numpy templating system
- #43: MAINT: update six.py to not use lazy loading.
- #49: Taking on API Issues
- #50: Add idwt
- #53: readme updated with info related to Py3 version
- #63: Remove six
- #65: Thresholding
- #70: MAINT: PEP8 fixes
- #72: BUG: fix \_downcoef for level > 1
- #73: MAINT: documentation and metadata update for repo fork
- #74: STY: fix pep8/pyflakes issues
- #77: MAINT: raise ValueError if data given to dwt or idwt is not 1D...

### 5.6.2 PyWavelets 0.4.0 Release Notes

#### Contents

- *PyWavelets 0.4.0 Release Notes*
  - *New features*
    - \* *1D and 2D inverse stationary wavelet transforms*
    - \* *Faster 2D and nD wavelet transforms*
    - \* *Complex floating point support*
    - \* *nD implementation of the multilevel DWT and IDWT*
    - \* *Wavelet transforms can be applied along a specific axis/axes*
    - \* *Example Datasets*
  - *Deprecated features*
  - *Backwards incompatible changes*
  - *Bugs Fixed*
  - *Other changes*
  - *Authors*
    - \* *Issues closed for v0.4.0*
    - \* *Pull requests for v0.4.0*

PyWavelets 0.4.0 is the culmination of 6 months of work. In addition to several new features, some changes and deprecations have been made to streamline the API.

This release requires Python 2.6, 2.7 or 3.3-3.5 and NumPy 1.6.2 or greater.

Highlights of this release include:

- 1D and 2D inverse stationary wavelet transforms
- Substantially faster 2D and nD discrete wavelet transforms
- Complex number support
- nD versions of the multilevel DWT and IDWT

## New features

### 1D and 2D inverse stationary wavelet transforms

1D (`iswt`) and 2D (`iswt2`) inverse stationary wavelet transforms were added. These currently only support even length inputs.

### Faster 2D and nD wavelet transforms

The multidimensional DWT and IDWT code was refactored and is now an order of magnitude faster than in previous releases. The following functions benefit: `dwt2`, `idwt2`, `dwtN`, `idwtN`.

### Complex floating point support

64 and 128-bit complex data types are now supported by all wavelet transforms.

### nD implementation of the multilevel DWT and IDWT

The existing 1D and 2D multilevel transforms were supplemented with an nD implementation.

### Wavelet transforms can be applied along a specific axis/axes

All wavelet transform functions now support explicit specification of the axis or axes upon which to perform the transform.

## Example Datasets

Two additional 2D grayscale images were added (*camera*, *ascent*). The previously existing 1D ECG data (*ecg*) and the 2D aerial image (*aero*) used in the demos can also now be imported via functions defined in `pywt.data` (e.g. `camera = pywt.data.camera()`)

## Deprecated features

A number of functions have been renamed, the old names are deprecated and will be removed in a future release:

- `intwave`, renamed to `integrate_wavelet`
- `centrfrq`, renamed to `central_frequency`
- `scal2frq`, renamed to `scale2frequency`
- `orthfilt`, renamed to `orthogonal_filter_bank`

Integration of general signals (i.e. not wavelets) with `integrate_wavelet` is deprecated.

The `MODES` object and its attributes are deprecated. The new name is `Modes`, and the attribute names are expanded:

- `zpd`, renamed to `zero`
- `cpd`, renamed to `constant`
- `sp1`, renamed to `smooth`
- `sym`, renamed to `symmetric`
- `ppd`, renamed to `periodic`
- `per`, renamed to `periodization`

### Backwards incompatible changes

`idwt` no longer takes a `correct_size` parameter. As a consequence, `idwt2` inputs must match exactly in length. For multilevel transforms, where arrays differing in size by one element may be produced, use the `waverec` functions from the `multilevel` module instead.

### Bugs Fixed

`float32` inputs were not always respected. All transforms now return `float32` outputs when called using `float32` inputs.

Incorrect detail coefficients were returned by `downcoef` when `level > 1`.

### Other changes

Much of the API documentation is now autogenerated from the corresponding function docstrings. The `numpydoc sphinx` extension is now needed to build the documentation.

### Authors

- Thomas Arildsen +
- François Boulogne
- Ralf Gommers
- Gregory R. Lee
- Michael Marino +
- Aaron O’Leary +
- Daniele Tricoli +
- Kai Wohlfahrt

A total of 8 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

### Issues closed for v0.4.0

- #46: Independent test comparison
- #95: Simplify Matlab tests
- #97: BUG: erroneous detail coefficients returned by downcoef with...
- #140: demo/dwt\_signal\_decomposition.py : TypeError: object of type...
- #141: Documentation needs update: ImportError: cannot import name 'multilevel'

### Pull requests for v0.4.0

- #55: [RFC] Api changes
- #59: Refactor convolution.c.src
- #64: MAINT: make LH, HL variable names in idwt2 consistent with dwt2
- #67: ENH: add wavedecn and waverecn functions
- #68: ENH: Faster dwtn and idwtn
- #88: DOC minor edit about possible naming
- #93: Added implementation of iswt and iswt2
- #98: fix downcoef detail coefficients for level > 1
- #99: complex support in all dwt and idwt related functions
- #100: replace mlabwrap with python-matlab-bridge in Matlab tests
- #102: Replace some .src expansion with macros
- #104: Faster idwtn/dwtn
- #106: make sure transforms respect float32 dtype
- #109: DOC: fix broken link in sidebar for html docs.
- #112: Complex fix
- #113: TST: don't build .exe installers on Appveyor anymore, only wheels.
- #116: [RFC] ENH: Add axis argument to dwt
- #117: MAINT: remove deprecated for loop syntax from Cython code
- #121: Fix typo
- #123: MAINT: remove some unused imports
- #124: switch travis from python 3.5-dev to 3.5
- #130: Add axis argument to multidim
- #138: WIP: Documentation updates for v0.4.0
- #139: Autogenerate function API docs
- #142: fix broken docstring examples in \_multilevel.py
- #143: handle None properly in waverec
- #144: Add importable images

- #145: DOC: Document MSVC versions

### 5.6.3 PyWavelets 0.5.0 Release Notes

#### Contents

- *PyWavelets 0.5.0 Release Notes*
  - *New features*
    - \* *1D Continuous Wavelet Transforms*
    - \* *New discrete wavelets*
    - \* *New extension mode: reflect*
    - \* *Multilevel DWT Coefficient Handling*
    - \* *More C function calls release the GIL*
    - \* *Multilevel wavelet transforms along specific axes*
    - \* *Faster multilevel stationary wavelet transforms*
  - *Deprecated features*
  - *Backwards incompatible changes*
  - *Bugs Fixed*
  - *Other changes*
  - *Authors*
    - \* *Issues closed for v0.5.0*
    - \* *Pull requests for v0.5.0*

PyWavelets is a Python toolbox implementing both discrete and continuous wavelet transforms (mathematical time-frequency transforms) with a wide range of built-in wavelets. C/Cython are used for the low-level routines, enabling high performance. Key Features of PyWavelets are:

- 1D, 2D and nD Forward and Inverse Discrete Wavelet Transform (DWT and IDWT)
- 1D, 2D and nD Multilevel DWT and IDWT
- 1D and 2D Forward and Inverse Stationary Wavelet Transform
- 1D and 2D Wavelet Packet decomposition and reconstruction
- 1D Continuous Wavelet Transform
- When multiple valid implementations are available, we have chosen to maintain consistency with MATLAB™'s Wavelet Toolbox.

PyWavelets 0.5.0 is the culmination of 1 year of work. In addition to several new features, substantial refactoring of the underlying C and Cython code have been made.

This release requires Python 2.6, 2.7 or 3.3-3.5 and NumPy 1.9.1 or greater. This will be the final release supporting Python 2.6 and 3.3.

Highlights of this release include:

- 1D continuous wavelet transforms

- new discrete wavelets added (additional Debauchies and Coiflet wavelets)
- new ‘reflect’ extension mode for discrete wavelet transforms
- faster performance for multilevel forward stationary wavelet transforms (SWT)
- n-dimensional support added to forward SWT
- routines to convert multilevel DWT coefficients to and from a single array
- axis support for multilevel DWT
- substantial refactoring/reorganization of the underlying C and Cython code

## New features

### 1D Continuous Wavelet Transforms

A wide range of continuous wavelets are now available. These include the following:

- Gaussian wavelets (`gaus1...` ‘‘gaus8’’)
- Mexican hat wavelet (`mexh`)
- Morlet wavelet (`mor1`)
- Complex Gaussian wavelets (`cgau1...` ‘‘cgau8’’)
- Shannon wavelet (`shan`)
- Frequency B-Spline wavelet (`fbsp`)
- Complex Morlet wavelet (`cmor`)

Also, see the new CWT-related demo: `demo/cwt_analysis.py`

### New discrete wavelets

Additional Debauchies wavelets (`db20...` ‘‘db38’’) and Coiflets (`coif6...` ‘‘coif17’’) have been added.

### New extension mode: reflect

Discrete wavelet transforms support a new extension mode, `reflect`. This mode pads an array symmetrically, but without repeating the edge value. As an example:

```
pad      array      pad
4 3 2 | 1 2 3 4 5 | 4 3 2
```

This differs from `symmetric`, which repeats the values at the boundaries:

```
pad      array      pad
3 2 1 | 1 2 3 4 5 | 5 4 3
```

## Multilevel DWT Coefficient Handling

New routines to convert the coefficients returned by multilevel DWT routines to and from a single n-dimensional array have been added. `pywt.coeffs_to_array` concatenates the output of `wavedec`, `wavedec2` or `wavedecn` into a single numpy array. `pywt.array_to_coefs` can be used to transform back from a single coefficient array to a format appropriate for `waverec`, `waverec2` or `waverecn`.

## More C function calls release the GIL

Cython code calling the wavelet filtering routines (DWT and SWT) now releases the global interpreter lock (GIL) where possible. A potential use case is in speeding up the batch computation of several large DWTs using multi-threading (e.g. via `concurrent.futures`).

## Multilevel wavelet transforms along specific axes

The axis specific transform support introduced in the prior release was extended to the multilevel DWT transforms. All `wavedec*` and `waverec*` routines have a new `axis` (1D) or `axes` (2D, nD) keyword argument. If unspecified the default behaviour is to transform all axes of the input.

## Faster multilevel stationary wavelet transforms

Stationary wavelet transforms are now faster when the number of levels is greater than one. The improvement can be very large (multiple orders of magnitude) for transforms with a large number of levels.

## Deprecated features

### Backwards incompatible changes

A `FutureWarning` was added to `swt2` and `iswt2` to warn about a pending backwards incompatible change to the order of the coefficients in the list returned by these routines. The actual change will not occur until the next release. Transform coefficients will be returned in descending rather than ascending order. This change is being made for consistency with all other existing multi-level transforms in PyWavelets.

### Bugs Fixed

`demo/image_blender.py` was updated to support the new api of Pillow 3.x

A bug related to size of assumed `size_t` on some platforms/compilers (e.g. Windows with mingw64) was fixed.

Fix to memory leak in `(i)dwt_axis`

Fix to a performance regression in `idwt` and `iswt` that was introduced in v0.4.0.

Fixed a bug in `dwt_n` and `idwt_n` for data with complex dtype when `axes != None`.

### Other changes

The minimum supported numpy version has been increased to 1.9.1.

Test coverage (including for the Cython and C code) via [Codecov](#) was added and the overall test coverage has been improved.

A substantial overhaul of the C extension code has been performed. Custom templating is no longer used. The intention is to make this code easier to maintain and expand in the future.

The Cython code has been split out into a multiple files to hopefully make relevant portions of the wrappers easier to find for future developers.

`setup.py` now relies on `setuptools` in all cases (rather than `distutils`).

## Authors

- Jonathan Dan +
- Ralf Gommers
- David Menéndez Hurtado
- Gregory R. Lee
- Holger Nahrstaedt +
- Daniel M. Pelt +
- Alexandre Saint +
- Scott Sievert +
- Kai Wohlfahrt
- Frank Yu +

A total of 10 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

## Issues closed for v0.5.0

- #48: Continuous wavelet transform?
- #127: Reorganize `_pywt`
- #160: Appveyor failing on recent PRs
- #163: Set up coveralls
- #166: Wavelet coefficients to single array (and vice versa?)
- #177: Fail to install `pywt` due to the use of `index_t` which conflict with the definition in `/usr/include/sys/types.h` on smartos `sysmte` (open solaris like system)
- #180: Memory leak
- #187: ‘reflect’ signal extension mode
- #189: bump minimum `numpy` version?
- #191: Upgrade removed Pillow methods
- #196: building in-place for development.
- #200: `swt` implementation is considerably slower than MATLAB
- #209: broken doctests
- #210: Run doctests in CI setup
- #211: Typo in `iswt` documentation

- #217: *blank\_discrete\_wavelet* does not properly initialize some properties
- #231: I can't compile pywt

### Pull requests for v0.5.0

- #148: Reorganize C v2
- #161: Remove numpy distutils
- #162: fix: iswt/idwt performance regression
- #164: Improved coefficients for db and coif
- #167: Add coverage (codecov.io)
- #168: convert transform coefficients to and from a single n-dimensional array
- #169: Remove templating
- #170: :Always install new pip on Appveyor
- #172: Adding of missing wavelets from the matlab list
- #178: use `Index_t` instead of `index_t`
- #179: add axis/axes support to multilevel discrete wavelet transforms
- #181: Fix memory leak
- #182: improve test coverage for `_multidim.py` and `_multilevel.py`
- #183: improve coverage for `_dwt.py`
- #184: fix corner case in `coeffs_to_array`
- #188: Drop GIL in `c_wt` calls
- #190: bump minimum numpy to 1.9
- #192: Upgrade to Pillow>=3 api
- #193: ENH: add 'reflect' extension mode
- #197: BLD: fix "python setup.py develop". Closes gh-196
- #198: Choose `clz*` based on `SIZE_MAX`
- #201: speedup multi-level swt
- #205: fix `dwtn/idwtn` with axes != None and complex data
- #206: DOC: correct typo in `iswt` docstring
- #207: minor documentation updates
- #208: document `coeff_to_array` and `array_to_coeff`
- #214: FIX: update several doctests to reflect the new wavelets added
- #218: FIX: initialize all properties of a blank discrete wavelet
- #219: document coordinate conventions for 2D DWT routines.
- #220: Run doctests on TravisCI
- #221: Documentation for `cwt` and `ContinuousWavelet`
- #222: consistent use of double backticks in docs

- #223: add FutureWarning about swt2 coefficient order
- #224: n-dimensional stationary wavelet transform (swtn) and axis support in swt, swt2
- #225: BUG: fix breakage on 32-bit Python.
- #226: DOC: update Copyright statements.
- #227: ENH: add kind keyword to wavelist()
- #228: MAINT: avoid using a builtin as variable name in qmf().
- #229: DOC: add swtn, iswt, iswt2 to the API documentation
- #230: add demo of batch processing via concurrent.futures
- #234: ENH: coeffs\_to\_array supports axes argument as recently added to wavedec\*
- #236: BLD: raise an ImportError if Cython should be installed but isn't.

## 5.6.4 PyWavelets 1.0.0 Release Notes

### Contents

- *PyWavelets 1.0.0 Release Notes*
  - *New features*
    - \* *New 1D test signals*
    - \* *C99 complex support*
    - \* *complex-valued CWT*
    - \* *More flexible specification of some continuous wavelets*
    - \* *Fully Separable Discrete Wavelet Transform*
    - \* *New thresholding methods*
    - \* *New anti-symmetric boundary modes*
    - \* *New functions to ravel and unravel wavedecn coefficients*
    - \* *New functions to determine multilevel DWT coefficient shapes and sizes*
  - *Deprecated features*
  - *Backwards incompatible changes*
  - *Bugs Fixed*
  - *Other changes*
  - *Authors*
    - \* *Issues closed for v1.0.0*
    - \* *Pull requests for v1.0.0*

We are very pleased to announce the release of PyWavelets 1.0. We view this version number as a milestone in the project's now more than a decade long history. It reflects that PyWavelets has stabilized over the past few years, and is now a mature package which a lot of other important packages depend on. A listing of those package won't be complete, but some we are aware of are:

- [scikit-image](#) - image processing in Python
- [imagehash](#) - perceptual image hashing
- [pyradiomics](#) - extraction of Radiomics features from 2D and 3D images and binary masks
- [tomopy](#) - Tomographic Reconstruction in Python
- [SpikeSort](#) - Spike sorting library implemented in Python/NumPy/PyTables
- [ODL](#) - operator discretization library

This release requires Python 2.7 or  $\geq 3.5$  and NumPy 1.9.1 or greater. The 1.0 release will be the last release supporting Python 2.7. It will be a Long Term Support (LTS) release, meaning that we will backport critical bug fixes to 1.0.x for as long as Python itself does so (i.e. until 1 Jan 2020).

### New features

#### New 1D test signals

Many common synthetic 1D test signals have been implemented in the new function `pywt.data.demo_signals` to encourage reproducible research. To get a list of the available signals, call `pywt.data.demo_signals('list')`. These signals have been validated to match the test signals of the same name from the [Wavelab](#) toolbox (with the kind permission of Dr. David Donoho).

#### C99 complex support

The Cython modules and underlying C library can now be built with C99 complex support when supported by the compiler. Doing so improves performance when running wavelet transforms on complex-valued data. On POSIX systems (Linux, Mac OS X), C99 complex support is enabled by default at build time. The user can set the environment variable `USE_C99_COMPLEX` to 0 or 1 to manually disable or enable C99 support at compile time.

#### complex-valued CWT

The continuous wavelet transform, `cwt`, now also accepts complex-valued data.

#### More flexible specification of some continuous wavelets

The continuous wavelets `"cmor"`, `"shan"` and `"fbsp"` now let the user specify attributes such as their center frequency and bandwidth that were previously fixed. See more on this in the section on deprecated features.

#### Fully Separable Discrete Wavelet Transform

A new variant of the multilevel n-dimensional DWT has been implemented. It is known as the fully separable wavelet transform (FSWT). The functions `fswavedecn` and `fswaverecn` correspond to the forward and inverse transforms, respectively. This differs from the existing `wavedecn` and `waverecn` in dimensions  $\geq 2$  in that all levels of decomposition are performed along a single axis prior to moving on to the next.

## New thresholding methods

`pywt.threshold` now supports non-negative Garotte thresholding (`mode='garotte'`). There is also a new function `pywt.threshold_firm` that implements firm (semi-soft) thresholding. Both of these new thresholding methods are intermediate between soft and hard thresholding.

## New anti-symmetric boundary modes

Two new boundary handling modes for the discrete wavelet transforms have been implemented. These correspond to whole-sample and half-sample anti-symmetric boundary conditions (`antisymmetric` and `antireflect`).

## New functions to ravel and unravel wavedecn coefficients

The function `ravel_coeffs` can be used to ravel all coefficients from `wavedec`, `wavedec2` or `wavedecn` into a single 1D array. Unraveling back into a list of individual n-dimensional coefficients can be performed by `unravel_coeffs`.

## New functions to determine multilevel DWT coefficient shapes and sizes

The new function `wavedecn_size` outputs the total number of coefficients that will be produced by a `wavedecn` decomposition. The function `wavedecn_shapes` returns full shape information for all coefficient arrays produced by `wavedecn`. These functions provide the size/shape information without having to explicitly compute a transform.

## Deprecated features

The continuous wavelets with names `"cmor"`, `"shan"` and `"fbsp"` should now be modified to include formerly hard-coded attributes such as their center frequency and bandwidth. Use of the bare names `"cmor"`, `"shan"` and `"fbsp"` is now deprecated. For `"cmor"` (and `"shan"`), the form of the wavelet name is now `"cmorB-C"` (`"shanB-C"`) where B and C are floats representing the bandwidth frequency and center frequency. For `"fbsp"` the form should now incorporate three floats as in `"fbspM-B-C"` where M is the spline order and B and C are the bandwidth and center frequencies.

## Backwards incompatible changes

Python 2.6, 3.3 and 3.4 are no longer supported.

The order of coefficients returned by `swt2` and input to `iswt2` have been reversed so that the decomposition levels are now returned in descending rather than ascending order. This makes these 2D stationary wavelet functions consistent with all of the other multilevel discrete transforms in PyWavelets.

For `wavedec`, `wavedec2` and `wavedecn`, the ability for the user to specify a `level` that is greater than the value returned by `dwt_max_level` has been restored. A `UserWarning` is raised instead of a `ValueError` in this case.

## Bugs Fixed

Assigning new data to the `Node` or `Node2D` no longer forces a cast to `float64` when the data is one of the other dtypes supported by the `dwt` (`float32`, `complex64`, `complex128`).

Calling `pywt.threshold` with `mode='soft'` now works properly for complex-valued inputs.

A segfault when running multiple `swt2` or `swtn` transforms concurrently has been fixed.

Several instances of deprecated numpy multi-indexing that caused warnings in numpy  $\geq 1.15$  have been resolved.

The 2d inverse stationary wavelet transform, *iswt2*, now supports non-square inputs (an unnecessary check for square inputs was removed).

Wavelet packets no longer convert float32 to float64 upon assignment to nodes.

Doctests have been updated to also work with NumPy  $\geq 1.14$ ,

Indexing conventions have been updated to avoid FutureWarnings in NumPy 1.15.

### Other changes

Python 3.7 is now officially supported.

### Authors

- 0-tree +
- Jacopo Antonello +
- Matthew Brett +
- Saket Choudhary +
- Michael V. DePalatis +
- Daniel Goertzen +
- Ralf Gommers
- Mark Harfouche +
- John Kirkham +
- Dawid Laszuk +
- Gregory R. Lee
- Michel Pelletier +
- Balint Reczey +
- SylvainLan +
- Daniele Tricoli
- Kai Wohlfahrt

A total of 16 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

### Issues closed for v1.0.0

The following 15 issues were closed for this release.

- #405: New warning appearing
- #397: Make pip install work if numpy is not yet installed
- #396: Allow more levels in wavedec
- #386: Improve documentation for cwt

- #396: Allow more levels in wavedec
- #368: Bug in ISWT2 for non-rectangular arrays
- #363: Crash threading swt2
- #357: reconstruction from array\_to\_coeff and waverec
- #352: FYI: PyWavelet does not correctly declare setup.py dependency...
- #338: upcoef - TypeError: No matching signature found
- #335: Build issue: PyWavelets does not install from sdist
- #333: user-friendly error messages regarding discrete vs. continuous...
- #326: Allow complex dtype of input
- #316: Test fail in some architectures
- #312: Documentation should suggest using the default conda channel
- #308: incorporate bandwidths into CWT wavelet names for families *cmor*,...
- #306: dwt\_max\_levels not enough documentation
- #302: Can't remove cA and then reconstruct
- #290: idwtm should treat coefficients set to None as zeros
- #288: RuntimeError and segfaults from swt2() in threaded environments

### Pull requests for v1.0.0

A total of 53 pull requests were merged for this release.

- #248: DOC: sync PyWavelets main descriptions.
- #249: Add pyqtgraph demo for plotting wavelets
- #254: DOC: fix rendering of wavelist docstring
- #255: ENH: improve iswt performance
- #256: ENH: add iswtm (n-dimensional inverse SWT)
- #257: s/addional/additional/
- #260: TST: test OS X build on TravisCI. Closes gh-75.
- #262: avoid some compiler warnings
- #263: MAINT: better exception message for Wavelet('continuous\_familyname')
- #264: add ASV (continued)
- #265: MAINT: fix more compiler warnings
- #269: allow string input in dwt\_max\_level
- #270: DOC: update ISWT documentation
- #272: allow separate wavelet/mode for each axis in routines based on...
- #273: fix non-integer index error
- #275: ENH: use single precision routines for half-precision inputs
- #276: update wp\_scalogram demo work with matplotlib 2.0

- #285: Fix spelling typo
- #286: MAINT: Package the license file
- #291: idwtm should allow coefficients to be set as None
- #292: MAINT: ensure tests are included in wheels
- #294: FIX: shape adjustment in waverec should not assume a transform...
- #299: DOC: update outdated scipy-user email address
- #300: ENH: compiling with C99 support (non-MSVC only)
- #303: DOC: better document how to handle omitted coefficients in multilevel...
- #309: Document how max levels are determined for multilevel DWT and...
- #310: parse CWT wavelet names for parameters
- #314: TST: Explicitly align data records in test\_byte\_offset()
- #317: TST: specify rtol and atol for assert\_allclose calls in test\_swt\_decomposition
- #320: Suggest using default conda channel to install
- #321: BLD: add pyproject.toml file (PEP 518 support).
- #322: support soft thresholding of complex valued data
- #331: Rename to CONTRIBUTING.rst
- #337: provide a more helpful error message for wrong wavelet type
- #339: check for wrong number of dimensions in upcoef and downcoef
- #340: DOC: fix broken link to Airspeed Velocity documentation
- #344: force legacy numpy repr for doctests
- #349: test case for CWT with complex input
- #350: better document the size requirements for swt/swt2/swtm
- #351: Add two new antisymmetric edge modes
- #353: DOC: add citation info to the front page of the docs.
- #354: add firm (semi-soft) and non-negative garotte thresholding
- #355: swt(): inference of level=None to depend on axis
- #356: fix: default level in *wavedec2* and *wavedecn* can be too conservative
- #360: fix Continuous spelling
- #361: AttributeError when using coeffs\_to\_array
- #362: Fix spelling of continuous globally
- #364: DOC: Explicitly print wavelet name for invalid wavelets
- #367: fix segfault related to parallel SWT
- #369: remove iswt2's restriction on non-square inputs
- #376: add common 1d synthetic signals
- #377: minor update to demo\_signals
- #378: numpy: 1.15 multiindexing warning. targetted fix

- #380: BLD: fix doc build on ReadTheDocs, need matplotlib for plots...
- #381: Fix corner case for small scales in CWT
- #382: avoid FutureWarnings related to multiindexing in Numpy1.15
- #383: adding Community guidelines
- #384: swap swt2 coefficient order (and remove FutureWarnings)
- #387: improve CWT docs
- #390: MAINT: update Python version support. Closes gh-385.
- #391: fix broken link in documentation
- #392: do not force float64 dtype on assignment to Node, Node2D
- #398: MAINT: update .gitignore for files generated during build.
- #401: Fix failing numpy 1.9.3 build on Travis CI
- #403: Change ValueError to UserWarning when level is > dwt\_max\_level
- #404: BLD: fix ReadTheDocs build. Outdated NumPy gave a conflict with MPL.
- #410: DOC: rewrite docs front page
- #413: add wavelets.pybytes.com disclaimer

## 5.6.5 PyWavelets 1.0.1 Release Notes

PyWavelets 1.0.1 is a bug-fix release with no new features compared to 1.0.0.

### Bugs Fixed

Key-based assignment of coefficients to a `FswavedecnResult` object (i.e. via its `__setitem__` method) has been fixed.

The order that the individual subband coefficients were stacked by the function `pywt.ravel_coeffs` is now guaranteed to be consistent across all supported Python versions. Explicit alphabetic ordering of subband coefficient names is used for consistent ordering regardless of Python version.

### Authors

- Gregory R. Lee

### Issues closed for v1.0.1

- #426: Ordering of the coefficients stacked in `pywt.ravel_coeffs` can vary across Python versions
- #425: error when trying to assign modified coefficients to a `FswavedecnResults` object

### Pull requests for v1.0.1

- #423: fix bug in `FswavedecnResult.__setitem__` and improve docstrings
- #427: fix: enforce consistent coefficient order for `ravel_coeffs`



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